# PROMETHEE METHODS:

# SENSIBILITY STUDY & REMARKS

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# INTRODUCTION AND OBJECTIVE

- In this presentation, the sensibility of PROMETHEE method to the use of different versions of independence property is studied;

- Rules and mathematical conditions upon which PROMETHEE keeps their original results is constructed;

- post-optimality studies and enquiries allowing to expect the new results and their values are proposed.

# PROMETHEE METHOD

- PROMETHEE method was introduced for the first time by Brans at 1982,
- An outranking method which take into account the generalized criteria,

<u>From the decision matrix A</u>

- The pairwise comparison is determined using the following preference index:

$$\pi(A_k, A_l) = \sum_{j=1}^{n} w_j p_j (A_k, A_l)$$

$p_j(A_k, A_l)$ is the preference function

# PROMETHEE METHOD

- The overall ranking uses the outgoing flux and the incoming:

$$\Phi^{+}(A_k) = \frac{1}{m-1} \sum_{a \in A \setminus \{A_k\}} \pi(A_k, a) \quad \& \quad \Phi^{-}(A_k) = \frac{1}{m-1} \sum_{a \in A \setminus \{A_k\}} \pi(a, A_k)$$

- An alternative $A_k$ outranks another alternative $A_l$ if:

$$\Phi^{+}(A_k) \geq \Phi^{+}(A_l) \text{ and } \Phi^{-}(A_k) \leq \Phi^{-}(A_l)$$

With at list one strict inequality

PROMETHEE I provides a partial ranking

# INDEPENDENCE PROPERTY VERSIONS

- Version 1: changing a non-optimal alternative $a$ by another worse one doesn't affect the overall ranking

<u>Formally</u>

Let be $g_i'(x) = g_i(x)$ *for all* $x \neq a$ *and* $g_i'(a) \leq g_i(a)$, *we have :*

$$G(A, g_1', \ldots, g_n')|A \setminus \{a\} = G(A, g_1, \ldots, g_n)|A \setminus \{a\}$$

- Version 2 (independence of non-discriminating elements):deleting an alternative $\hat{a}$ doesn't change the overall ranking

<u>Formally</u>

$\forall\, a \neq \hat{a}$, we have $aJ\hat{a}$, or $aI\hat{a}$, or $aP\hat{a}$, or $\hat{a}Pa$, then

$$g(S\ /\ (A \setminus \{\hat{a}\})) = \mathrm{g}(S)\ /\ (A \setminus \{\hat{a}\})$$

# Independence property versions

- Version 3 (independence of the best or the worst ranked elements): deleting the best (resp. the worst) alternative $\hat{a}$ doesn't change the overall ranking

<u>Formally</u>

$\forall\, a \neq \hat{a}$, we have $ag(S)\hat{a}$, or $\hat{a}g(S)a$, then

$$g(S \,/\, (A \setminus \{\hat{a}\})) = \mathrm{g(S)} \,/\, (A \setminus \{\hat{a}\})$$

- Version 4 (independence of the best or the worst ranked elements): deleting the group of best (resp. the worst) alternatives $B$ doesn't change the overall ranking

<u>Formally</u>

if $\mathrm{B} \subset A$, and if $\forall b \in B, \forall a \in A \setminus B$: $ag(S)b$ and $b\overline{g(S)}a$, or $\mathrm{b}g(S)a$ and $a\overline{g(S)}b$ then

$$g(S \,/\, (A \setminus B)) = \mathrm{g(S)} \,/\, (A \setminus B)$$

# INDEPENDENCE PROPERTY VERSIONS

- Version 1: changing a non-optimal alternative *a* by another worse one doesn't affect the overall ranking

<u>Formally</u>

Let be $g_i'(x) = g_i(x)$ *for all* $x \neq a$ *and* $g_i'(a) \leq g_i(a)$, *we have :*

$$G(A, g_1', \ldots, g_n')|A \setminus \{a\} = G(A, g_1, \ldots, g_n)|A \setminus \{a\}$$

# Version I

- An indifference relation in PROMETHEE I outranking remains unchanged iff:

$$D_k^+ = D_l^+ \quad \text{and} \quad D_k^- = D_l^-$$

where

$$D_i^+ = \pi(A_i, A'_p) - \pi(A_i, A_p)$$

$$D_i^- = \pi(A_p, A_i) - \pi(A'_p, A_i)$$

<u>Example</u>: the overall ranking of the following data is:

$$A_1 \boldsymbol{I} A_3 P A_2.$$

# Version 1

- Performance matrix

Performance matrix

| Actions \ Criteria | $j = 1$ | $j = 2$ | $j = 3$ |
|---|---|---|---|
| $A_1$ | 3 | 3 | 1 |
| $A_2$ | 3 | 2 | 1 |
| $A_3$ | 3 | 2 | 2 |

- Introducing new alternative $A_2' = (2, 1, 1)$ changes the overall ranking to $A_1 P A_3 P A_2'$ indeed,

Characteristics parameters

| | $A_3$ | $A_1$ |
|---|---|---|
| $D^+$ | 1/3 | 0 |
| $D^-$ | 0 | 0 |

# Version I

- A preference relation in PROMETHEE I outranking is conserved iff:

$$D_k^+ \geq D_l^+ - [\Phi^+(a_k) - \Phi^+(a_l)]$$

$$D_k^- \leq D_l^- - [\Phi^-(a_l) - \Phi^-(a_k)]$$

where

$$D_i^+ = \pi(A_i, A_p') - \pi(A_i, A_p)$$

$$D_i^- = \pi(A_p, A_i) - \pi(A_p', A_i)$$

# Independence property versions

Version 3 (independence of the best or the worst ranked elements):

deleting the best (resp. the worst) alternative $\hat{a}$ doesn't change the overall ranking

Formally

$\forall\ a \neq \hat{a}$, we have $a g(S) \hat{a}$, or $\hat{a} g(S) a$, then

$$g(S\ /\ (A \setminus \{\hat{a}\})) = \mathrm{g}(\mathrm{S})\ /\ (A \setminus \{\hat{a}\})$$

# Version 3

- An indifference relation :

$$\pi(A_k, \hat{a}) = \pi(A_l, \hat{a}) \text{ \underline{And} } \pi(\hat{a}, A_k) = \pi(\hat{a}, A_l)$$

- A preference relation :

$$(m-1)(\Phi^+(A_k)\text{-}\Phi^+(A_l)) \geq \pi(A_k, \hat{a}) - \pi(A_l, \hat{a})$$

And

$$(m-1)(\ \Phi^-(A_k)\text{-}\Phi^-(A_l)) \leq \pi(\hat{a}, A_k) - \pi(\hat{a}, A_l)$$

- An incomparability :

$$(m-1)(\Phi^+(A_k)\text{-}\Phi^+(A_l)) \geq (or \leq) \pi(A_k, \hat{a}) - \pi(A_l, \hat{a})$$

And

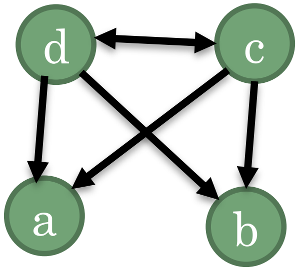
$$(m-1)(\Phi^-(A_k)\text{-}\Phi^-(A_l)) \geq (or \leq) \pi(\hat{a}, A_k) - \pi(\hat{a}, A_l)$$

# Version 3

- <u>Example</u>: consider the following data

| | J=1 | J=2 | J=3 |
|---|---|---|---|
| a | 1 | 1 | 3 |
| d | 3 | 3 | 1 |
| b | 3 | 2 | 1 |
| c | 3 | 2 | 2 |

- the overall ranking is: c$IdPbJa$




- Deleting the best alternative $d$ provides new ranking between $a$ and $b$:

$$cPbPa$$

# Version 3

Indeed,

| $\Phi^+(\boldsymbol{b})$-$\Phi^+(\boldsymbol{a})$ | $\pi(\boldsymbol{b},\boldsymbol{d}) - \pi(\boldsymbol{a},\boldsymbol{d})$ |
|---|---|
| -1/9 | -1/9 |
| $\Phi^-(\boldsymbol{b})$-$\Phi^-(\boldsymbol{a})$ | $\pi(\boldsymbol{d},\boldsymbol{b}) - \pi(\boldsymbol{d},\boldsymbol{a})$ |
| -3/9 | -1/9 |

Deleting the best alternative $d$ provides the following inequalities:

$$\Phi^+(\boldsymbol{b})\text{-}\Phi^+(\boldsymbol{a}) = 1/3(\pi(\boldsymbol{b},\boldsymbol{d}) - \pi(\boldsymbol{a},\boldsymbol{d}))$$

and

$$\Phi^-(\boldsymbol{b})\text{-}\Phi^-(\boldsymbol{a}) \leq 1/3(\pi(\boldsymbol{d},\boldsymbol{b}) - \pi(\boldsymbol{d},\boldsymbol{a}))$$

So, $bPa$

# Independence property versions

- Version 4 (independence of the best or the worst ranked elements):

deleting the group of best (resp. the worst) alternatives $B$ doesn't change the overall ranking

<u>Formally</u>

if B $\subset A$, and if $\forall b \in B, \forall a \in A \setminus B$: $ag(S)b$ and $b\overline{g(S)}a$, or b$g(S)a$ and $a\overline{g(S)}b$ then

$$g(S\ /\ (A \setminus B)) = \mathrm{g}(S)\ /\ (A \setminus B)$$

# Version 4

- An indifference relation :

$$\sum_{\hat{a} \in B} \pi(A_k, \hat{a}) = \sum_{\hat{a} \in B} \pi(A_l, \hat{a})$$

<u>And</u>

$$\sum_{\hat{a} \in B} \pi(\hat{a}, A_k) = \sum_{\hat{a} \in B} \pi(\hat{a}, A_l)$$

- A preference relation :

$$(m-1)(\ \Phi^+(A_k)\text{-}\Phi^+(A_l)) \geq \sum_{\hat{a} \in B} \pi(A_k, \hat{a}) - \sum_{\hat{a} \in B} \pi(A_l, \hat{a})$$

And

$$(m-1)(\ \Phi^-(A_k)\text{-}\Phi^-(A_l)) \leq \sum_{\hat{a} \in B} \pi(\hat{a}, A_k) - \sum_{\hat{a} \in B} \pi(\hat{a}, A_l)$$

- An incomparability :

$$(m-1)(\ \Phi^+(A_k)\text{-}\Phi^+(A_l)) \geq (or \leq) \sum_{\hat{a} \in B} \pi(A_k, \hat{a}) - \sum_{\hat{a} \in B} \pi(A_l, \hat{a})$$

And

$$(m-1)(\Phi^-(A_k)\text{-}\Phi^-(A_l)) \geq (or \leq) \sum_{\hat{a} \in B} \pi(\hat{a}, A_k) - \sum_{\hat{a} \in B} \pi(\hat{a}, A_l)$$

# VERSION 4

- From the example of version 3, deleting the set of best alternatives $B = \{d, c\}$, provides the following comparison:

| $\Phi^+(\boldsymbol{b})$-$\Phi^+(\boldsymbol{a})$ | $\sum_{\hat{a}\in B} \pi(\boldsymbol{b}, \hat{a}) - \sum_{\hat{a}\in B} \pi(\boldsymbol{a}, \hat{a})$ |
|---|---|
| -1/9 | -2/9 |
| $\Phi^-(\boldsymbol{b})$-$\Phi^-(\boldsymbol{a})$ | $\sum_{\hat{a}\in B} \pi(\hat{a}, b) - \sum_{\hat{a}\in B} \pi(\hat{a}, a)$ |
| -3/9 | -2/9 |

Deleting the set B provides the following inequalities:

$$\Phi^+(\boldsymbol{b})\text{-}\Phi^+(\boldsymbol{a}) \geq 1/3\left(\sum_{\hat{a}\in B} \pi(\boldsymbol{b}, \hat{a}) - \sum_{\hat{a}\in B} \pi(\boldsymbol{a}, \hat{a})\right)$$

and

$$\Phi^-(\boldsymbol{b})\text{-}\Phi^-(\boldsymbol{a}) \leq 1/3\left(\sum_{\hat{a}\in B} \pi(\hat{a}, b) - \sum_{\hat{a}\in B} \pi(\hat{a}, a)\right)$$

So, $bPa$

## REMARKS

- PROMETHEE I doesn’t verify any version of the discussed properties,
- Version 2 has same conditions as in version 3,
- It is easy to find examples which show that PROMETHEE I doesn’t verify version 2 of independence,
- Indifference relation is the most sensitive to any change or delete of data,

# REFERENCES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