PROMETHEE

METHODS

ULB

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Solvay Brussels School
Economics & Management

Preference Modeling with the PROMETHEE and GAIA Methods.

National Taipei University
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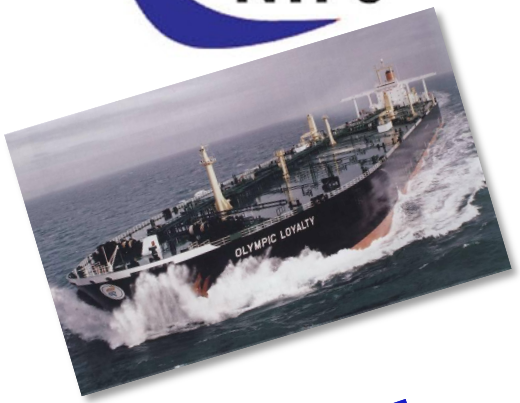
Summary

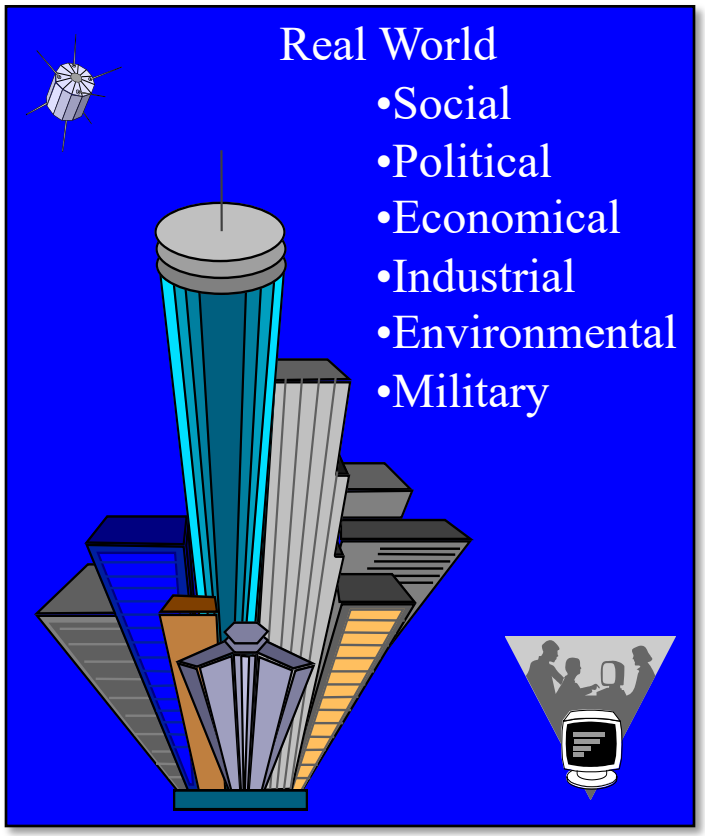
- Decisions
- Decision Aid
- Multicriteria Decision Aid vs Optimization
- PROMETHEE & GAIA
- Preference Modeling
- Visual PROMETHEE Software

Decisions

- Personal decisions
  - Choose a restaurant tonight
  - Choose a university
  - Purchase a new phone, a new car, …
- Business decisions
  - Develop a new product
  - Choose a computer system
  - Investments, strategies, project management, …
- Political decisions
  - Join the EU… Leave the EU…
  - Build a new hospital
  - Regional investment, taxes, …



Decision Making

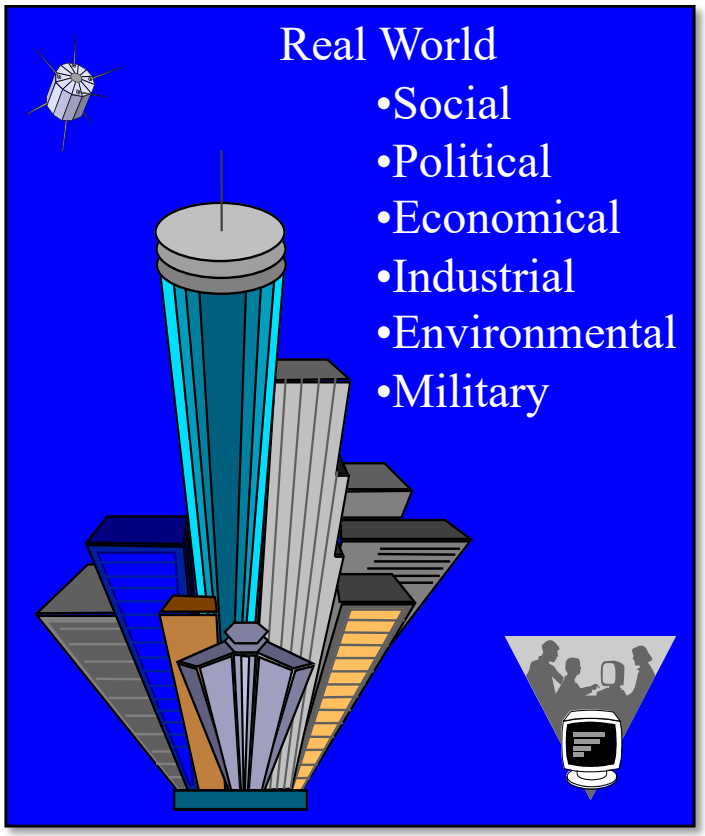


- Describe,
- Understand,
- Manage.

2 Approaches :

- Qualitative approach,
- Quantitative approach.

Decision Aid



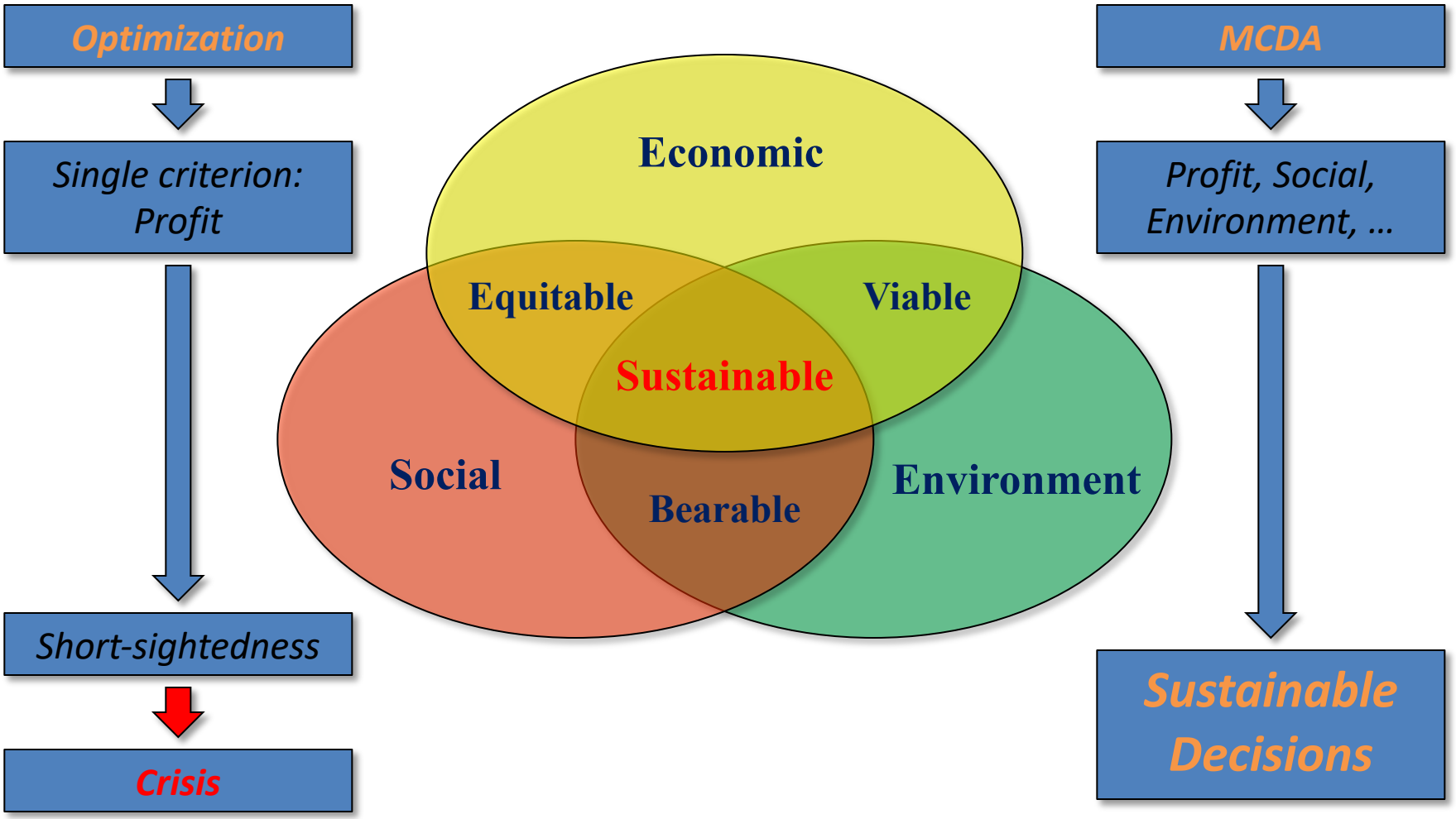
Quantitative model

- Approximation to real world!
- ➢ Decision Aid.

Quantitative Model?

- Describe possible decisions (actions)?
  - List, variables, …
- Objective? Best decision? Best choice?
  - Minimize costs?
  - Maximize profit?
  - Maximize quality?
  - Minimize impacts?
- Optimization models?
- Or MCDA (MultiCriteria Decision Aid) models…

MCDA vs Optimization



Some Examples: Decision or Evaluation Problems

- Locating a new plant, a new shop, ...
- Human resources management.
- Purchasing equipment.
- Assessing the quality of suppliers.
- Evaluating projects.
- Selecting an investment strategy.
- Making political decisions…

Multicriteria Table

- Actions:
  - Possible decisions,
  - items to evaluate.
- Criteria:
  - quantitative,
  - qualitative.

Multicriteria Table

Action 1	
Action 2	
Action 3	
Action 4	
Action 5	
…	

Multicriteria Table

	Crit. 1 (unit)	Crit. 2 (unit)	Crit. 3 (unit)	Crit. 4 (unit)	…
Action 1					
Action 2					
Action 3					
Action 4					
Action 5					
…					

Multicriteria Table

	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	…
Action 1	18	135	G	Yes	…
Action 2	9	147	B	Yes	…
Action 3	15	129	VG	No	…
Action 4	12	146	VB	?	…
Action 5	7	121	G	Yes	…
…	…	…	…	…	…

Plant Location

	Investment (M€)	Costs (k€)	Environm. (impact)	…
Site 1	18	135	G	…
Site 2	9	147	B	…
Site 3	15	129	VG	…
Site 4	12	146	VB	…
Site 5	7	121	G	…
…	…	…	…	…

Purchase Options

	Price (€)	Reliability (days)	Maintenance (estimate)	…
Product A	18	135	G	…
Product B	9	147	B	…
Product C	15	129	VG	…
Product D	12	146	VB	…
Product E	7	121	G	…
…	…	…	…	…

A Simple Example

The purchase of a new car by a Greek family

Objectives:

- Economy (price),
- Usage (fuel consumption),
- Performance (power),
- Space,
- Comfort.

Multicriteria Table

Cars	Price	Power	Fuel	Space	Comfort
Tourism A	26.000 €	75	8,0	average	average
Sport	29.000 €	110	9,0	very bad	bad
Tourism B	25.500 €	85	7,0	good	average
Luxury 1	38.000 €	90	8,5	good	very good
Economic	15.000 €	50	7,5	bad	very bad
Luxury 2	35.000 €	85	9,0	very good	good

- Best buy?



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- Best buy?





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- Best buy?
- Best compromise ?





Multicriteria Table

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- Best buy?
- Best compromise?
- Priorities of the buyer?

How to address a decision problem?

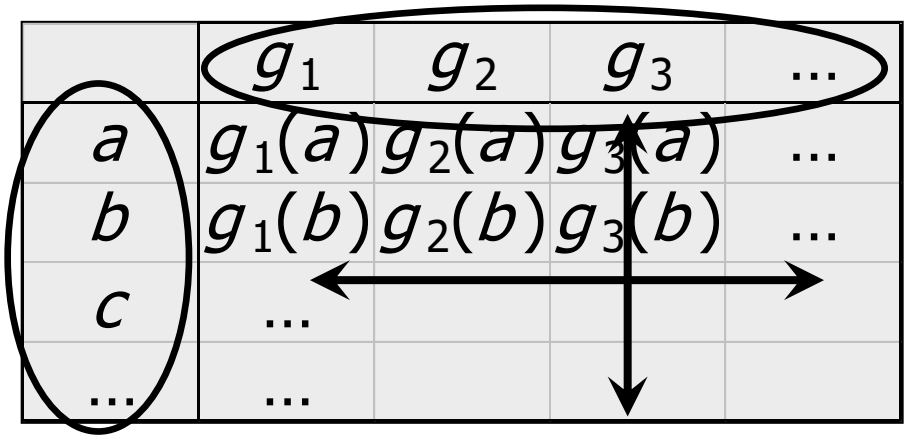
- What is the problem?
- What are the issues?
- Who is involved (stakeholders)?
- What is possible (decisions)?
- What is at stake (objectives)?
- How to measure achievement (criteria)?

➢ **Modeling…**

Modeling… 1… 2… 3…

2.
Define the
criteria

1.
Define the
actions



3.
Model
preferences

Qualitative vs Quantitative Criteria

- Quantitative criteria:
  - Natural numerical scale.
- Qualitative criteria:
  - Qualitative ordinal scale (ex: Likert scales).
  - Maximum 9 levels (7 ± 2) to ensure a consistent evaluation.
  - Presence of a neutral level?
  - Examples:
    - Very good, Good, Average, Bad, Very bad
    - Yes, No
    - ++, +, 0, -, --
    - ++, +, -, --
  - Underlying numerical scale (coding).

Common approaches

- Weighted sum
  - Completely compensatory (averaging).
  - Scale normalization problems.
  - Loss of information:
    - Multicriteria → unicriterion optimization.
- MAUT (multi-attribute utility theory)
  - Still completely compensatory logic.
  - Utility function:
    - Existence?
    - Building procedure?

Outranking Methods

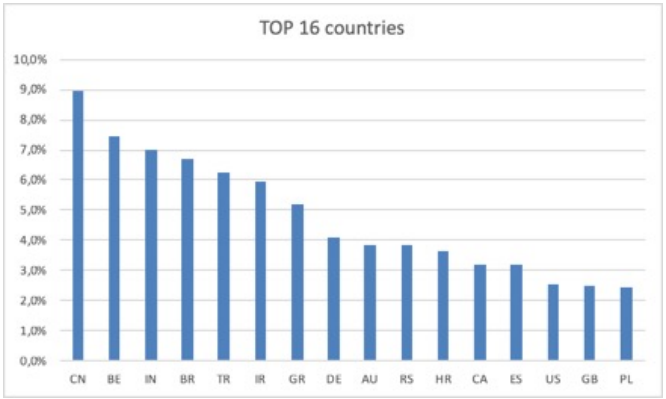
- Majority principle.
- Pairwise comparison of actions.
- Closer to the decision problem.
- Partially compensatory approach.
- **ELECTRE** methods (1968-).
- **PROMETHEE & GAIA** methods (1982-).

Why **PROMETHEE**?

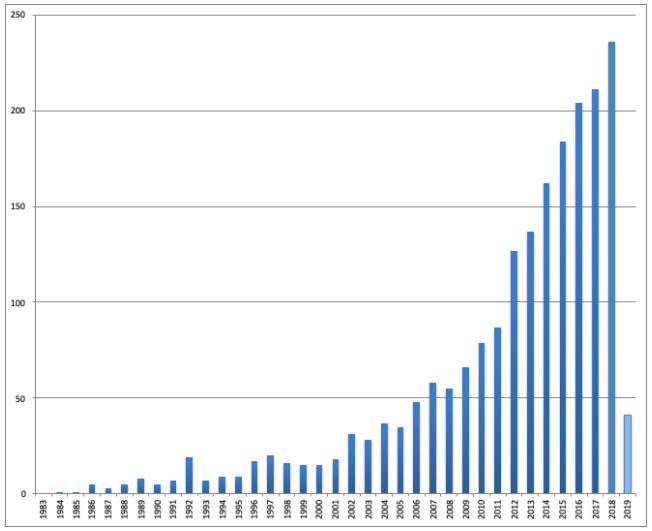
- Proven methodology:
  - 30+ years development,
  - Over 2000 published scientific papers.
- « Simplicity ».
- Visual tools.
- Sensitivity analysis tools.
- Interactivity.
- **Visual PROMETHEE** software.

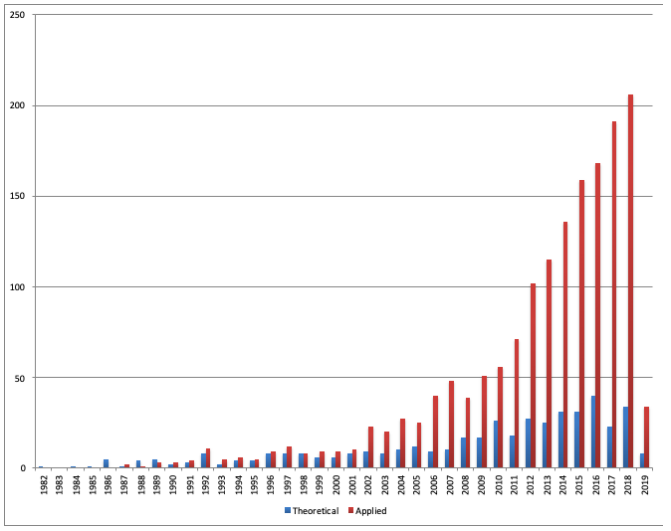
Some statistics…

- First paper presented in 1982 by J-P. Brans.
- Over 2000 published papers as of today.
- 31 papers published by 37 authors from Taiwan, from 1992 to 2019.
  (worldwide #20, after PRC, Belgium, India, Brazil, Turkey, Iran, Greece, Germany, Australia, Serbia, Croatia, Canada, Spain, USA, UK, Poland, … ; #14 PPM)
- Main fields of application:
  - Environment
  - Industry
  - Services / Public sector
  - Energy
  - Finance



PROMETHEE Timeline





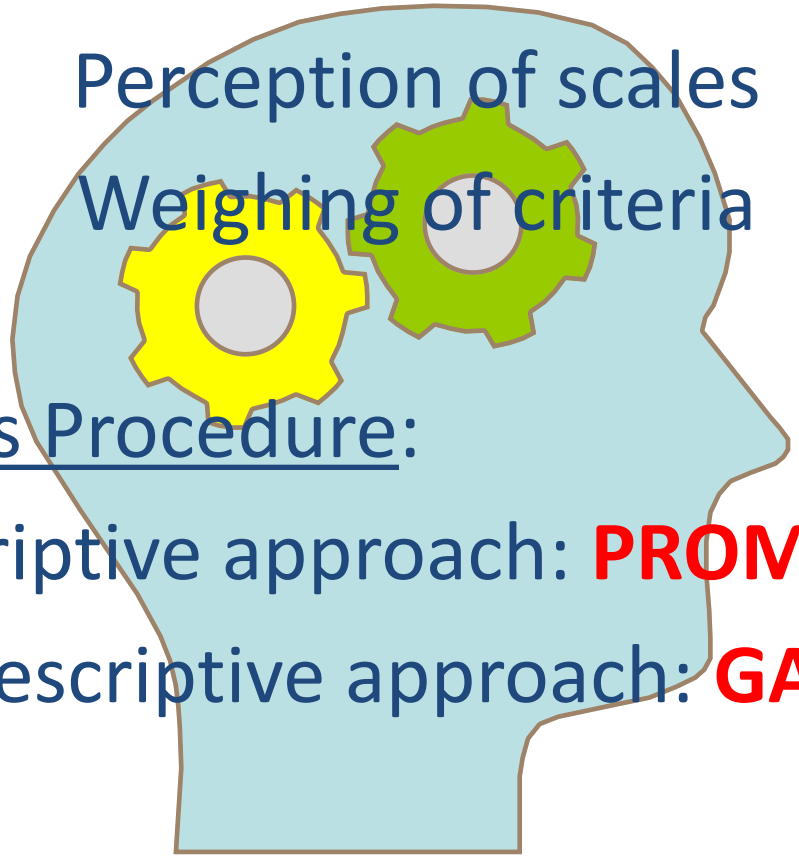
- Over 2000 papers published:
  - 80% applied – 20% theoretical
  - 57% related to "societal" fields
- Median year: 2014
- Over 3600 authors from 83 countries.

PROMETHEE Methods

- Preference modelling:

  Perception of scales

  Weighing of criteria



- Analysis Procedure:

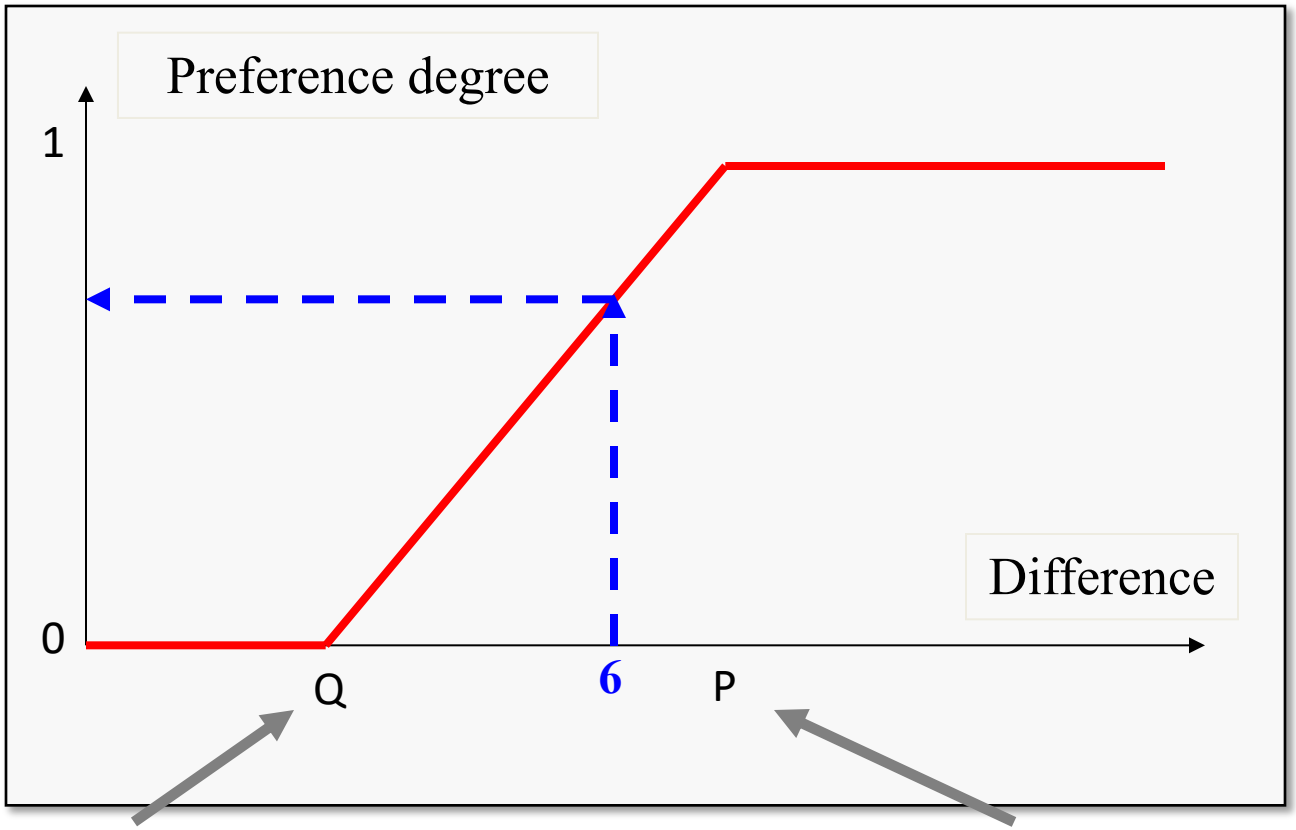
  Prescriptive approach: **PROMETHEE**

  Descriptive approach: **GAIA**

Comparison of 2 Actions

	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	…
Action 1	18	135	G	Yes	…
Action 2	9	147	Difference = 6		…
Action 3	15	129	VG	No	…
Action 4	12	146	VB	?	…
Action 5	7	121	G	Yes	…
…	…	…	…	…	…

Preference Function



Indifference threshold

Linear

Preference threshold

Pairwise Comparisons

- For each criterion g_j :
  - Preference function P_j
  - Weight w_j
- Multicriteria preference degree of a over b :

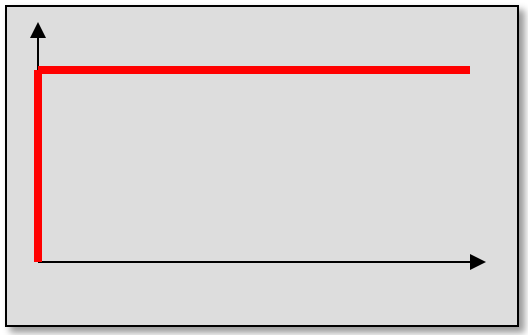
$$\pi(a,b) = \sum_{j=1}^{k} w_j \, P_j(a,b)$$

  - Properties:

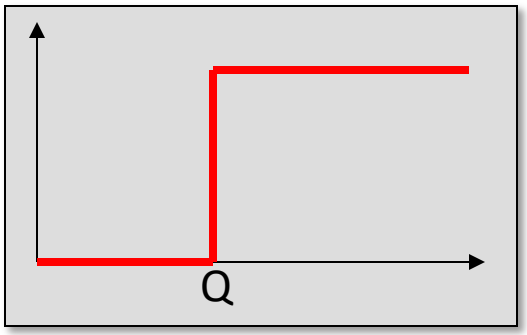
$$0 \leq \pi(a,b) \leq 1$$

$$\pi(a,b) > 0 \Rightarrow \pi(b,a) = 0$$

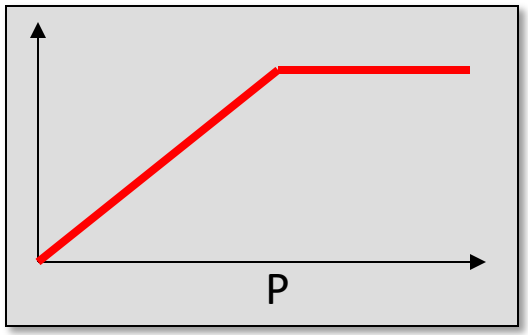
Preference Functions
(as in **Visual PROMETHEE** software)



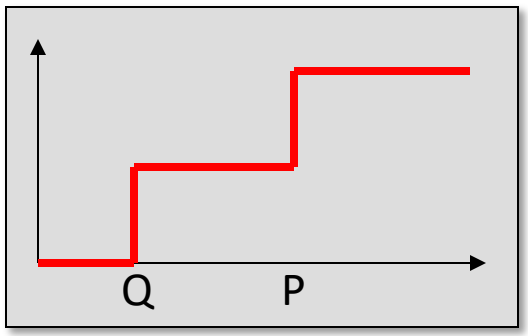
Usual



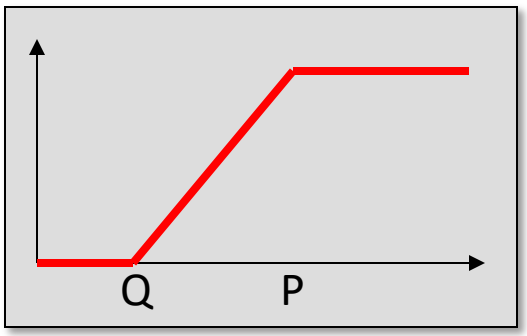
U-shape



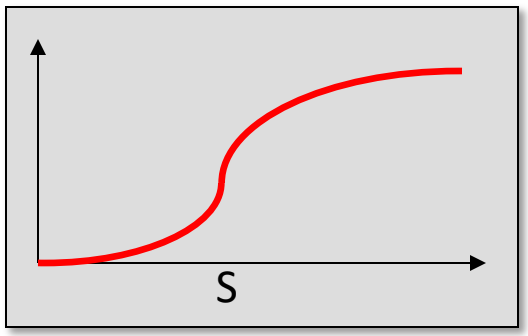
V-shape



Level



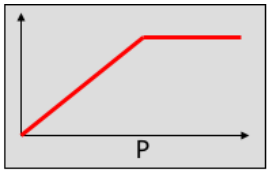
Linear



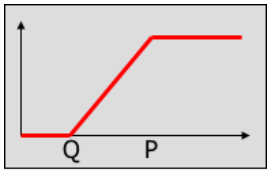
Gaussian

Preference Functions

- For continuous quantitative criteria (e.g. cost, price, power):
  - V-shape (no indifference threshold)
  - Linear

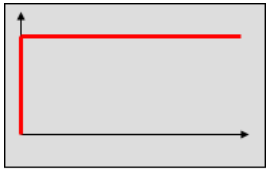


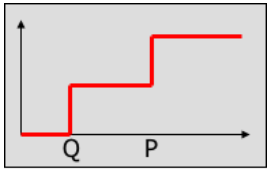
« V » shape



Linear

- For qualitative or discrete quantitative criteria (e.g. « very good to very bad », number of USB ports):
  - Usual (no thresholds)
  - Level


Usual



Level

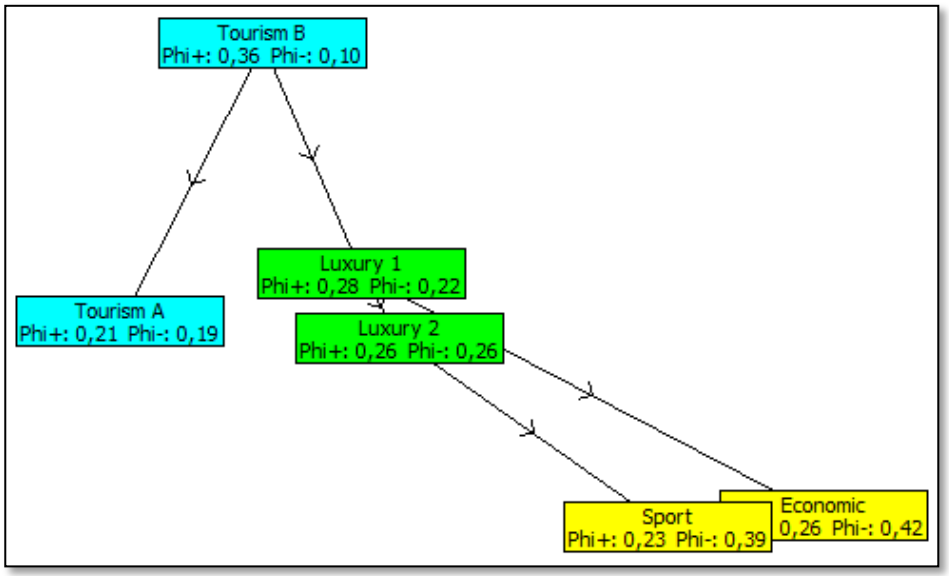
Preference Flows

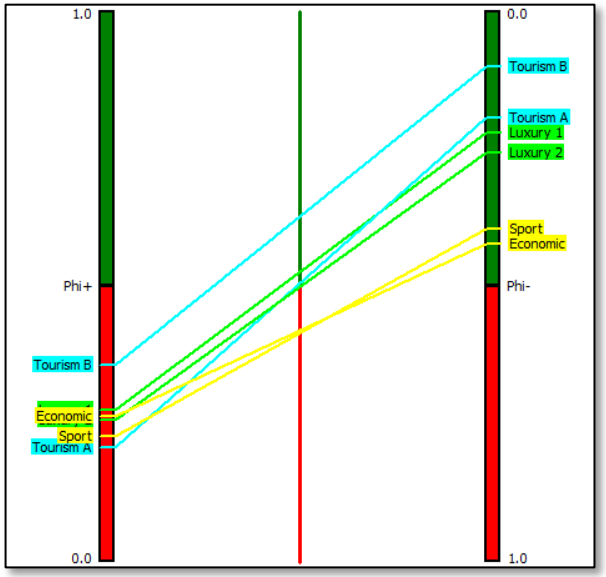
- Relative scores computed for the actions.
- Leaving (+) and entering (–) flows:
  - Strength: $\mathbf{0} \leq \phi^{+} \leq \mathbf{1}$
  - Weakness: $\mathbf{0} \leq \phi^{-} \leq \mathbf{1}$
- Net flow:
  - Balance: $\mathbf{-1} \leq \phi = \phi^{+} - \phi^{-} \leq \mathbf{+1}$
- Unicriterion net flows:
  - Standardized scores for each criterion:

$$\text{criterion } f_j \quad \Rightarrow \quad \mathbf{-1} \leq \phi_j \leq \mathbf{+1}$$

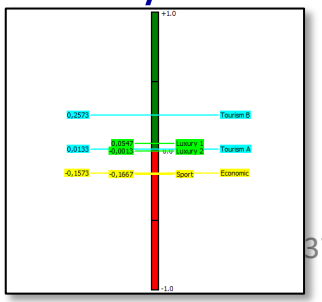
PROMETHEE I & II Rankings

- **PROMETHEE I** : partial ranking – ϕ^+, ϕ^-





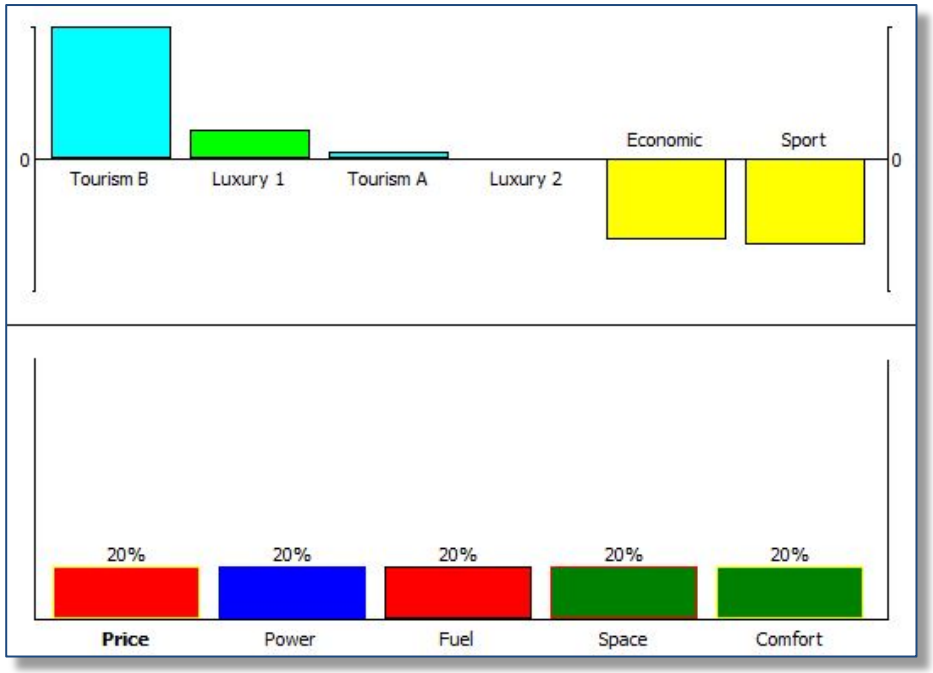
- **PROMETHEE II** : complete ranking – ϕ

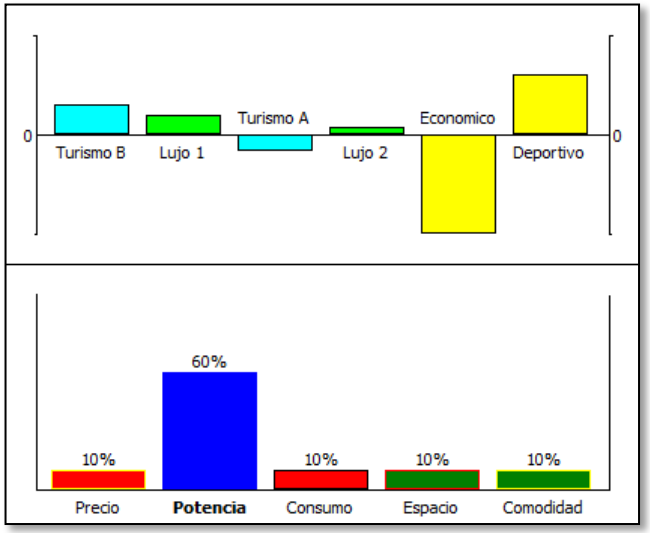


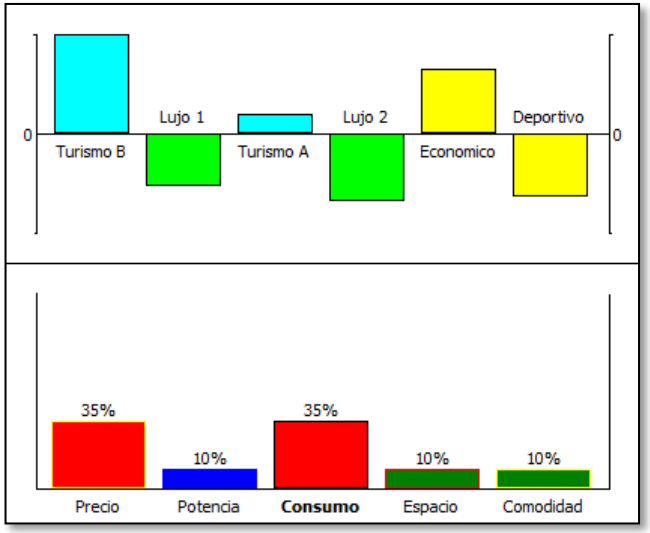
Sensitivity Analysis with **PROMETHEE**

- Criteria weights ↔ **PROMETHEE** rankings.
- Interactive weight sensitivity analysis:
  « Walking Weights ».
- Robustness with respect to weight values?
  - Weight stability intervals.
  - Visual weight stability intervals.

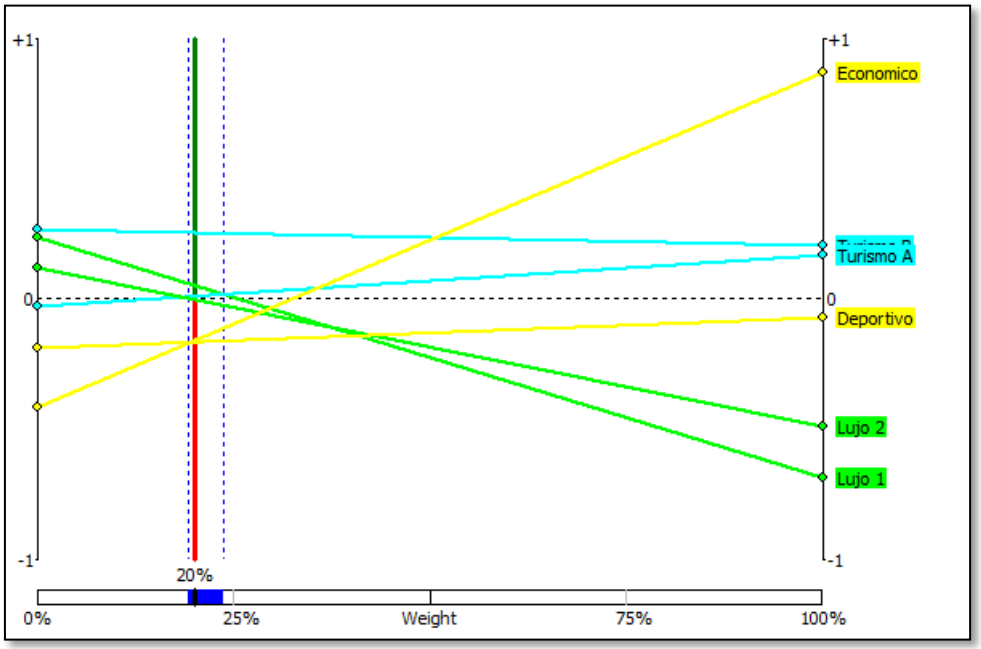
Walking Weights



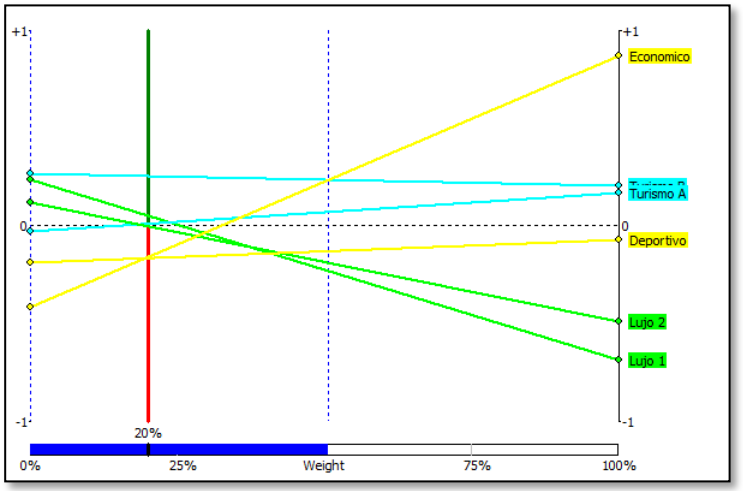




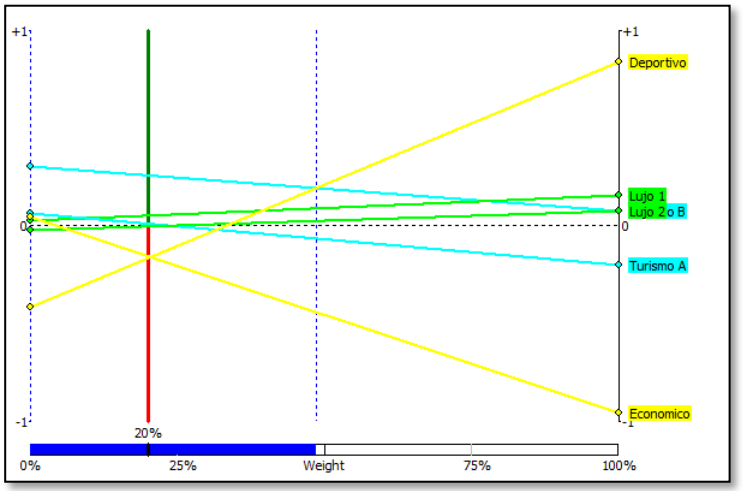
Visual Stability Intervals



VSI for « Price » (level 6):
[19.20% , 23.70%]



VSI for « Price » (level 1): [0.00% , 50.68%]



VSI for « Power » (level 1): [0.00% , 48.65%]

Limits of a Ranking Method

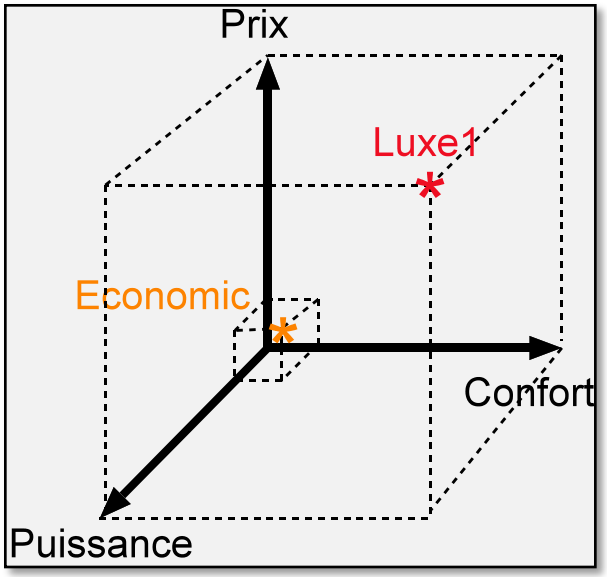
- Robustness of the ranking?
- « Blind » sensitivity analysis.
- Closely ranked actions can have quite different profiles.
- Origin of incomparabilities?

➢ Usefulness of a complementary descriptive approach.

GAIA

- Visual descriptive analysis.
- Better understanding of the decision problem:
  - Conflicting criteria.
  - Action profiles.
  - Possible compromise solutions.
- Reducing the multicriteria dimension:
  - Principal components analysis.

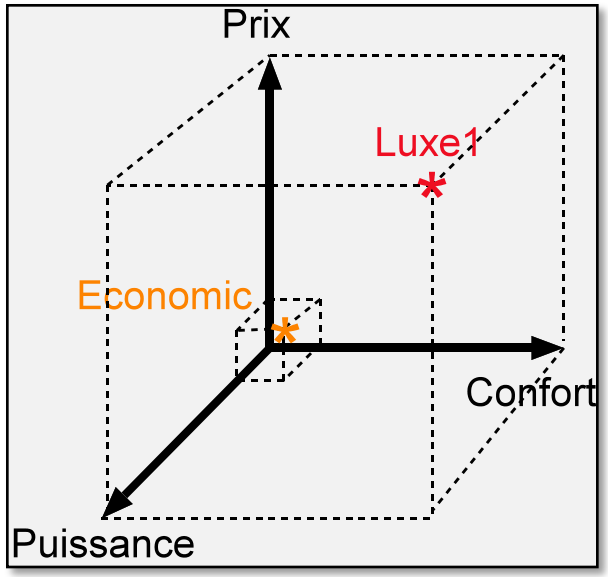
GAIA

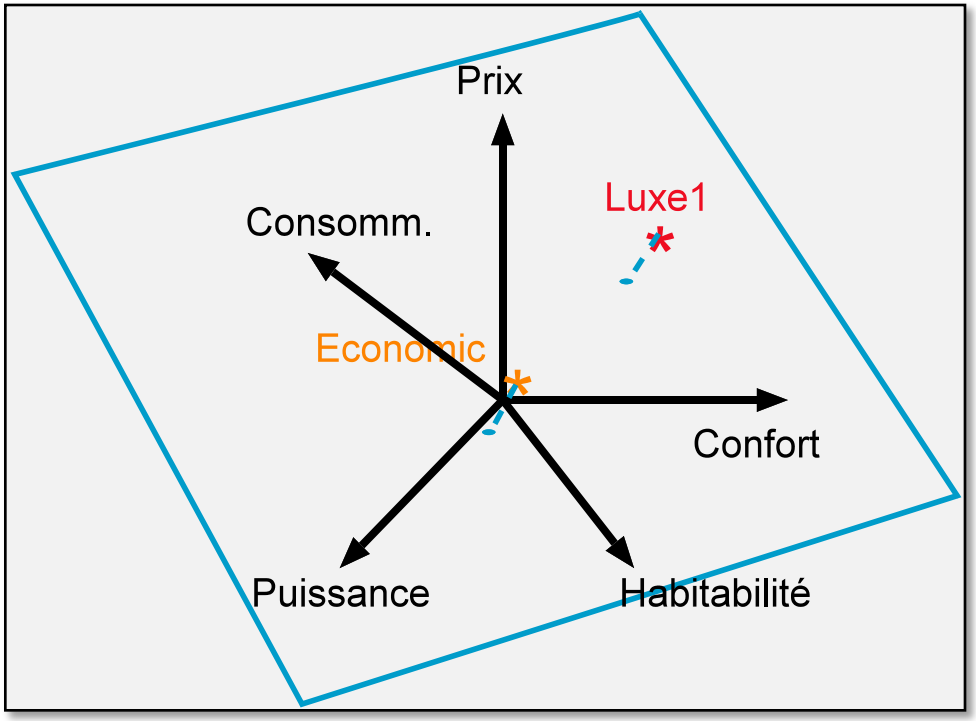


- Graphical representation.
- 5 dimensions!

GAIA

1. **Computation of unicriterion net flows (normalization)**
2. **Projection on a plane:**

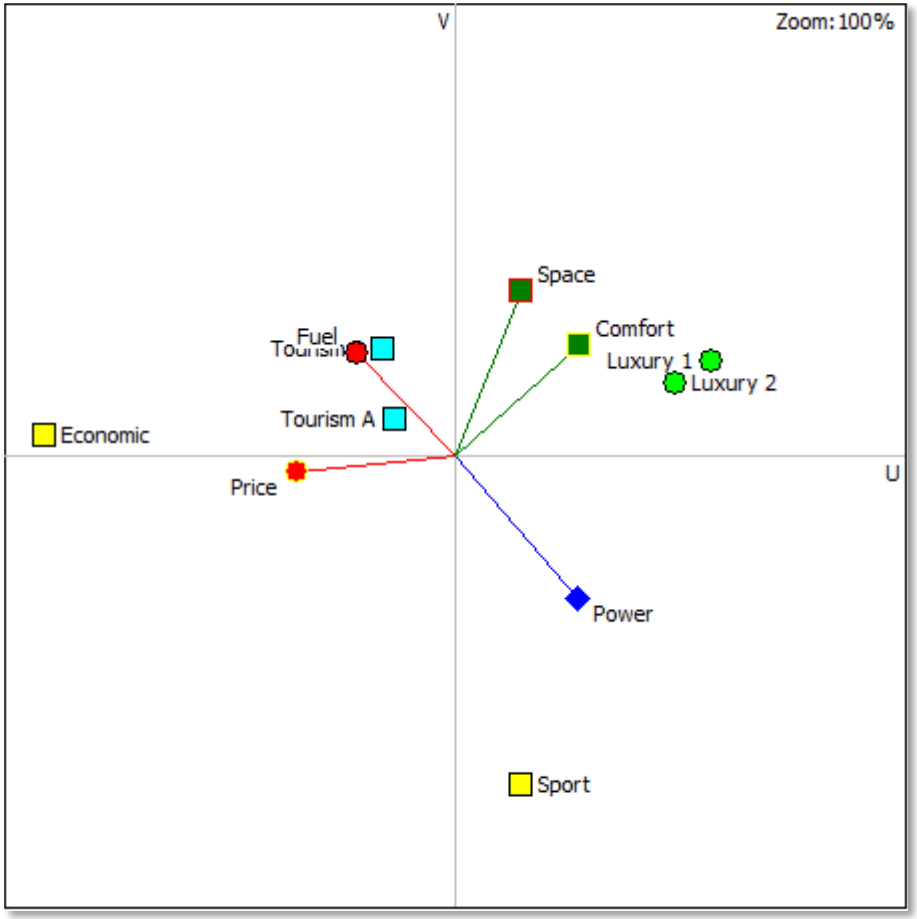




- Graphical representation.
- 5 dimensions!

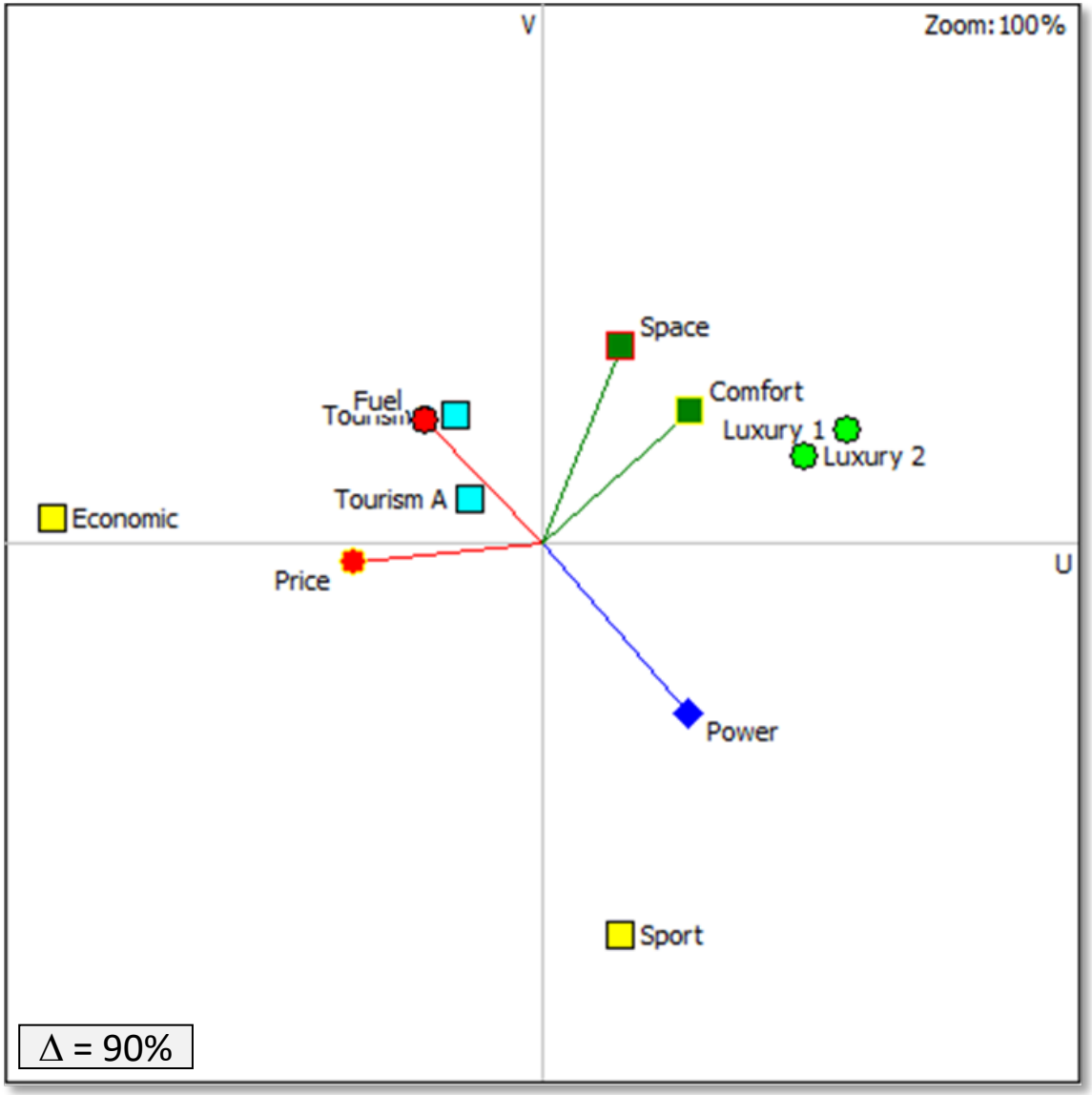
GAIA

- Discover conflicts among criteria.
- Identify potential compromises.
- Help to fix priorities.



GAIA

- Actions: points
- Criteria: axes

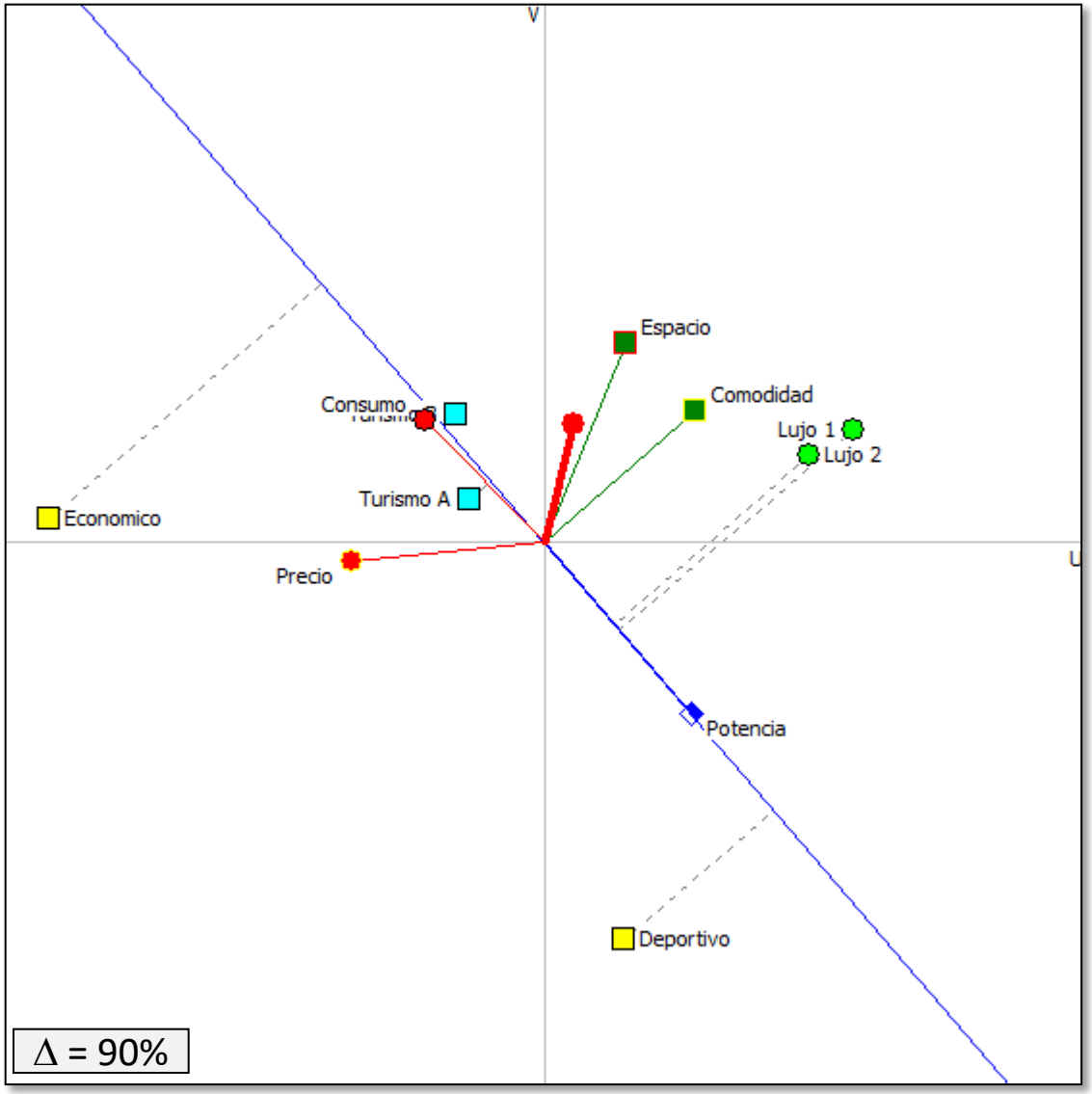


GAIA

Price

- Economic: 15 k€
- Tourism: 25,5-26 k€
- Sport: 29 k€
- Luxury: 35-38 k€

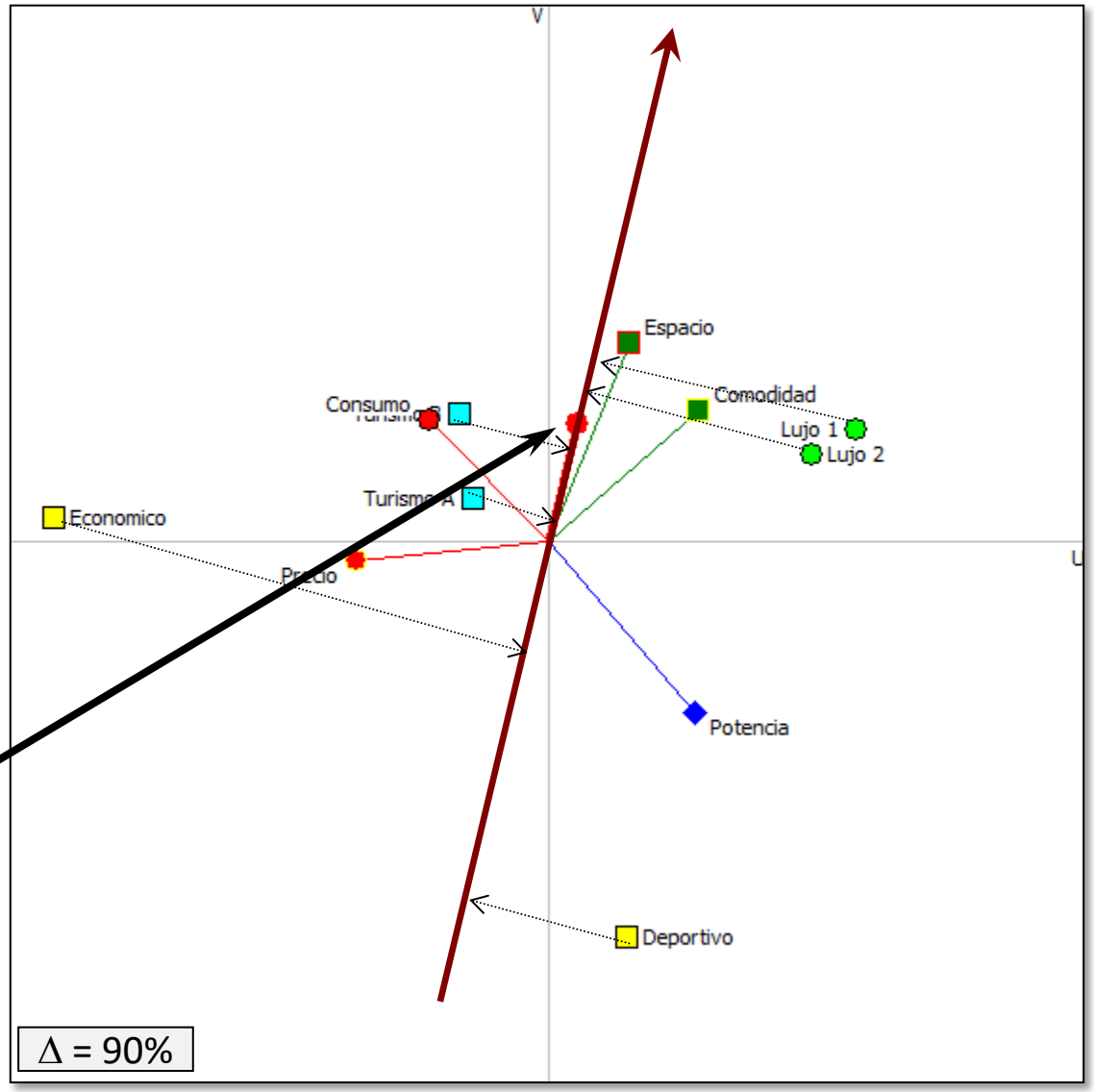
GAIA



Power

- Sport: 110 kW
- Luxury: 85-90 kW
- Tourism: 75-85 kW
- Economic: 50 kW

GAIA

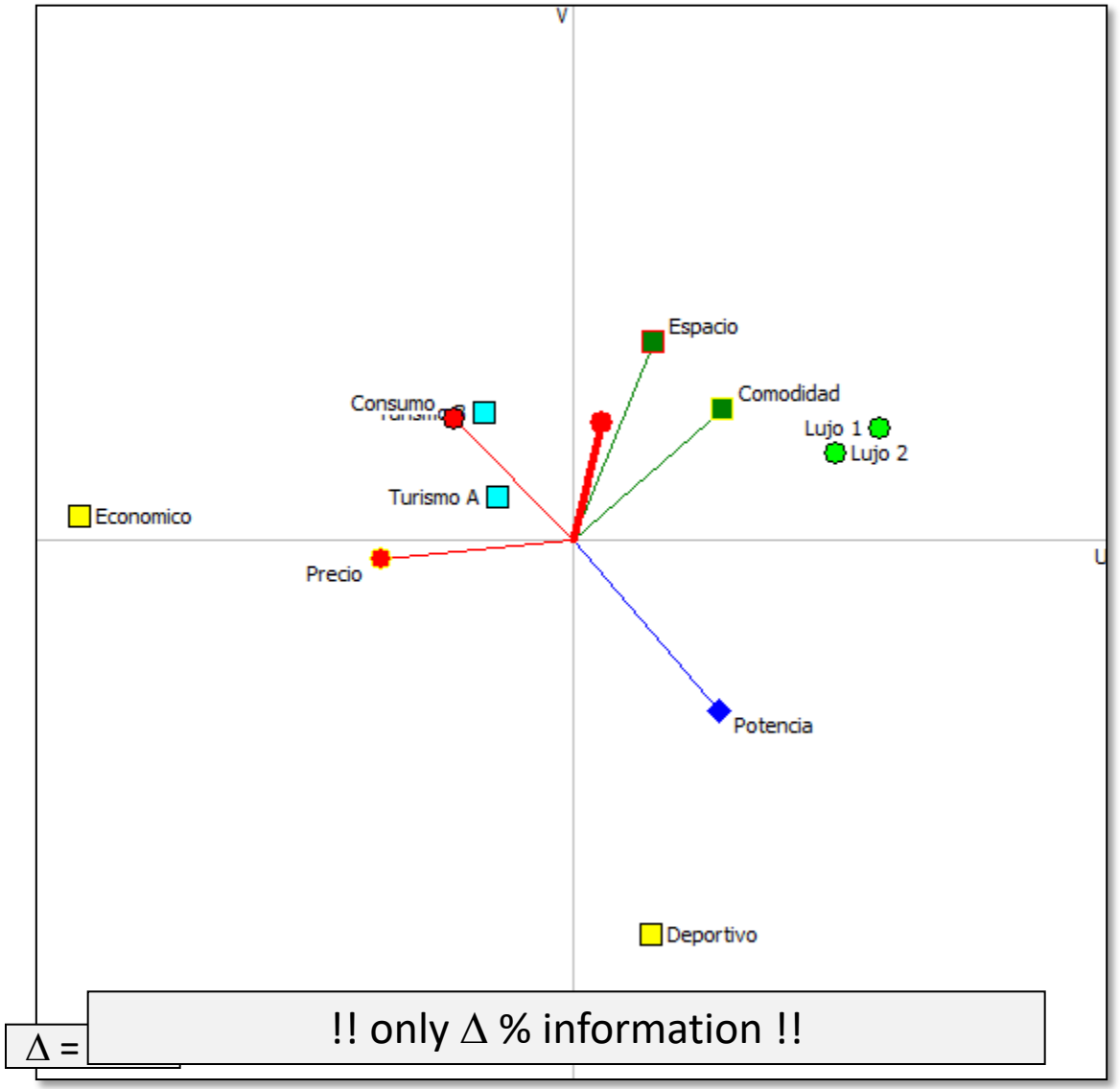


PROMETHEE II !

- Tourism B : 0,26
- Luxury 1 : 0,06
- Tourism A : 0,02
- Luxury 2 : 0,00
- Economic : -0,15
- Sport : -0,17

GAIA

- Actions: points
- Criteria: axes
- Decision axis



What is important in **GAIA**?

- Quality (Δ) of the representation:

  $\Delta < 60\%$ **$60\% \leq \Delta \leq 75\%$** **$\Delta > 75\%$**

- Direction of the criteria axes:
  - Groups of criteria expressing similar preferences (potential redundancies…).
  - Conflicting criteria.
- Length of the criteria axes:
  - Shorter: less discriminant criterion.
  - Longer: more discriminant criterion.
- Position of the actions:
  - Clusters of actions with similar profiles.
  - Position with respect to the **direction** of (not the proximity to) the criteria axes : strengths and weaknesses.

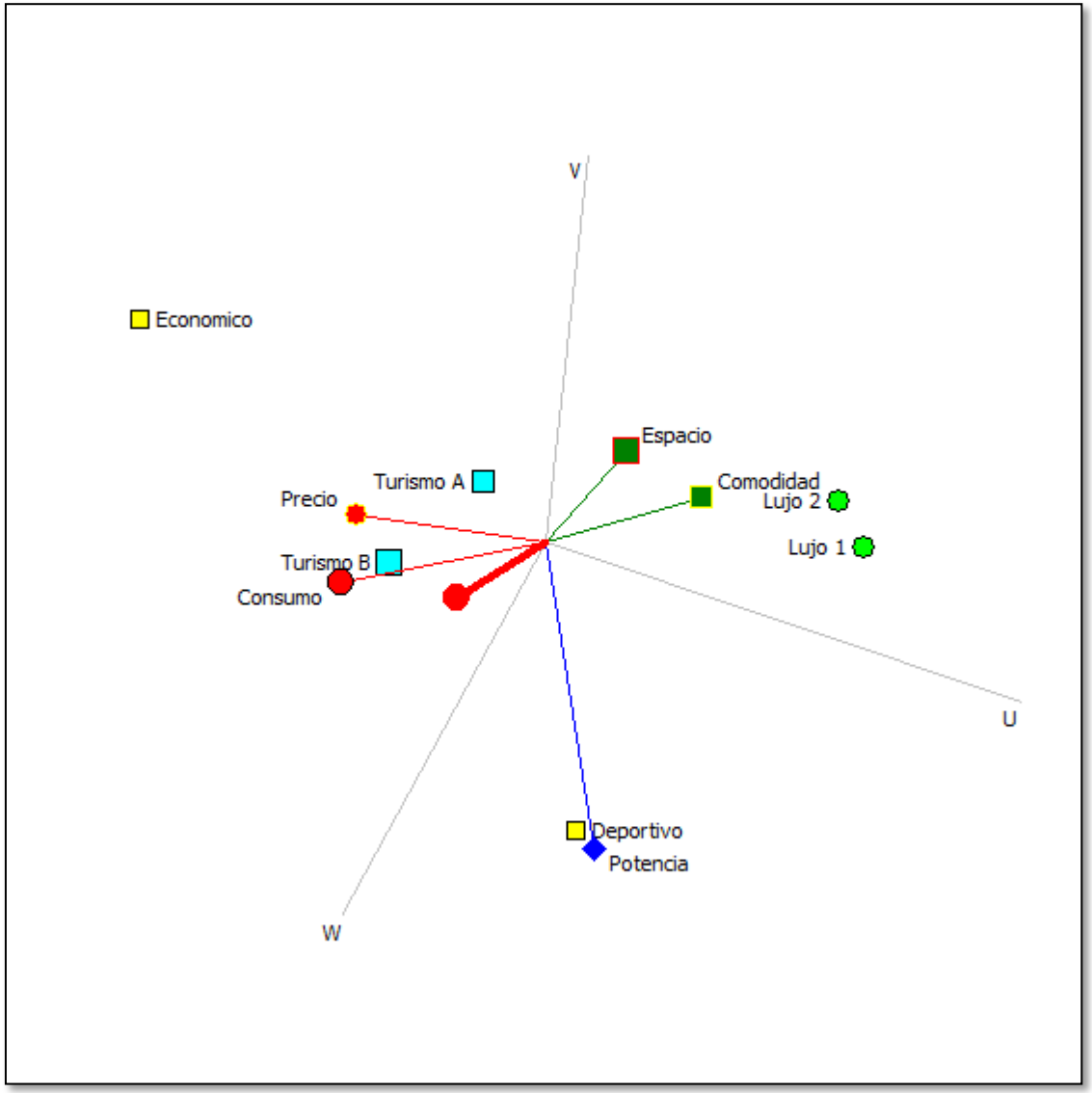
Sensitivity Analysis with **GAIA**

- Criteria weights ↔ Decision axis position.
- Interactive weight sensitivity analysis:
  « Walking Weights ».
- Robustness with respect to weight values?
  - Decision maker « brain » (**PROMETHEE VI**).
  - Area determined by the tip of the decision axis when criteria weights are changed within predefined percentages.

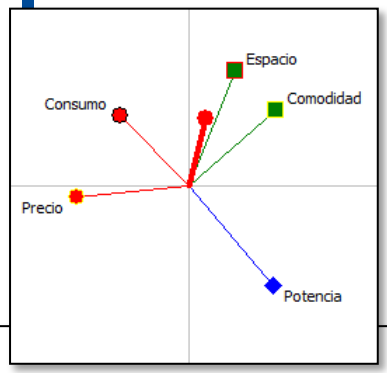
Enhancing **GAIA**

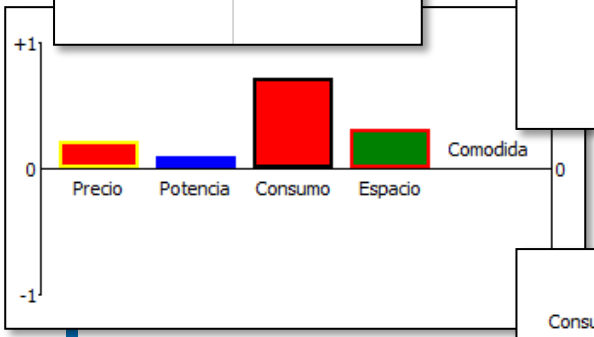
- Limits of **GAIA**:
  - Imperfect view of the multicriteria data (Δ%).
  - Potential distortion of the action profiles.
  - Non-optimal representation of the decision axis (weights) and distortion of the **PROMETHEE II** ranking (especially when the decision axis is shorter).
- New« **GAIA**-type » views:
  - **GAIA** 3D
  - **GAIA** Webs

GAIA 3D

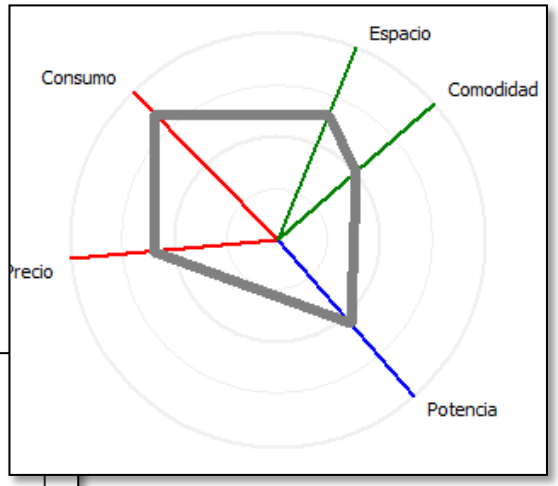

V
Economico
Espacio
Turismo A
Precio
Comodidad
Lujo 2
Lujo 1
Turismo B
Consumo
U
Deportivo
Potencia
W

GAIA Webs

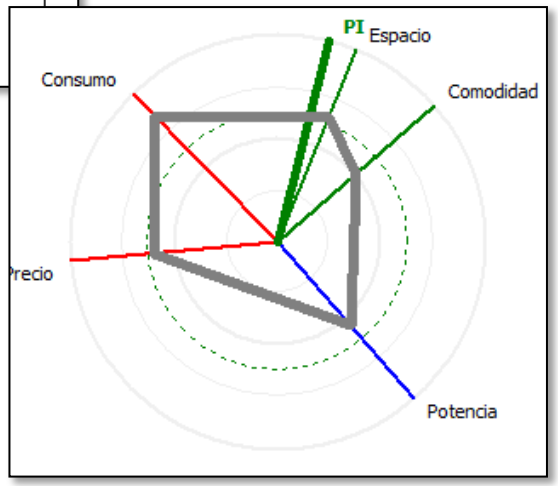


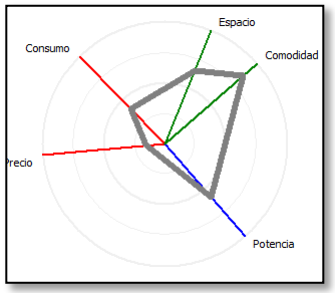


Action profile – Tourism B

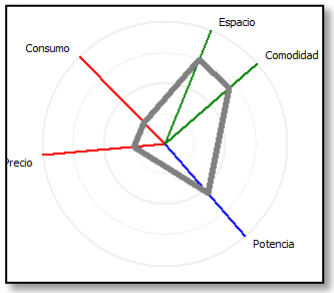


GAIA Web – Tourism B

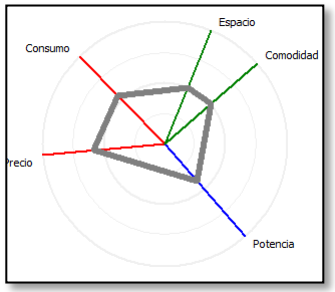




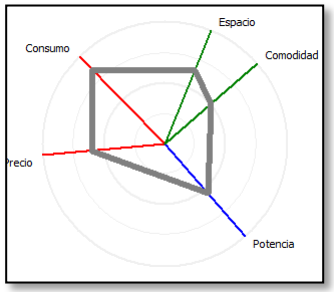
Luxury 1



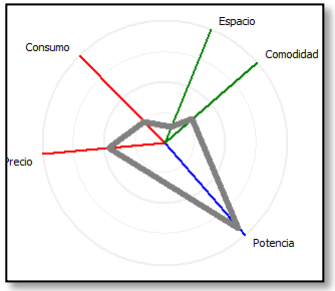
Luxury 2



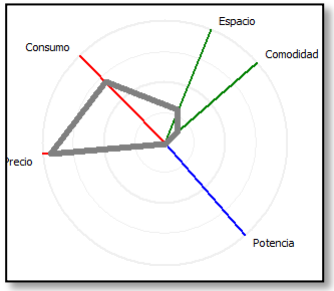
Tourism A



Tourism B



Sport



Economic

One or Several Stakeholders ?

- Single stakeholder:
  - One actor (the decision maker).
  - One multicriteria table and one preference structure.
- Multiple stakeholders:
  - Several actors (including decision maker(s)).
  - Several multicriteria tables and preference structures.
  - Search for consensus.

Example

- A Greek family.
- Three stakeholders (“decision makers”):
  - dad,
  - mom,
  - the kid.
- Three scenarios.
- Three multicriteria tables:
  - Different priorities.
  - Subjective evaluation of comfort.

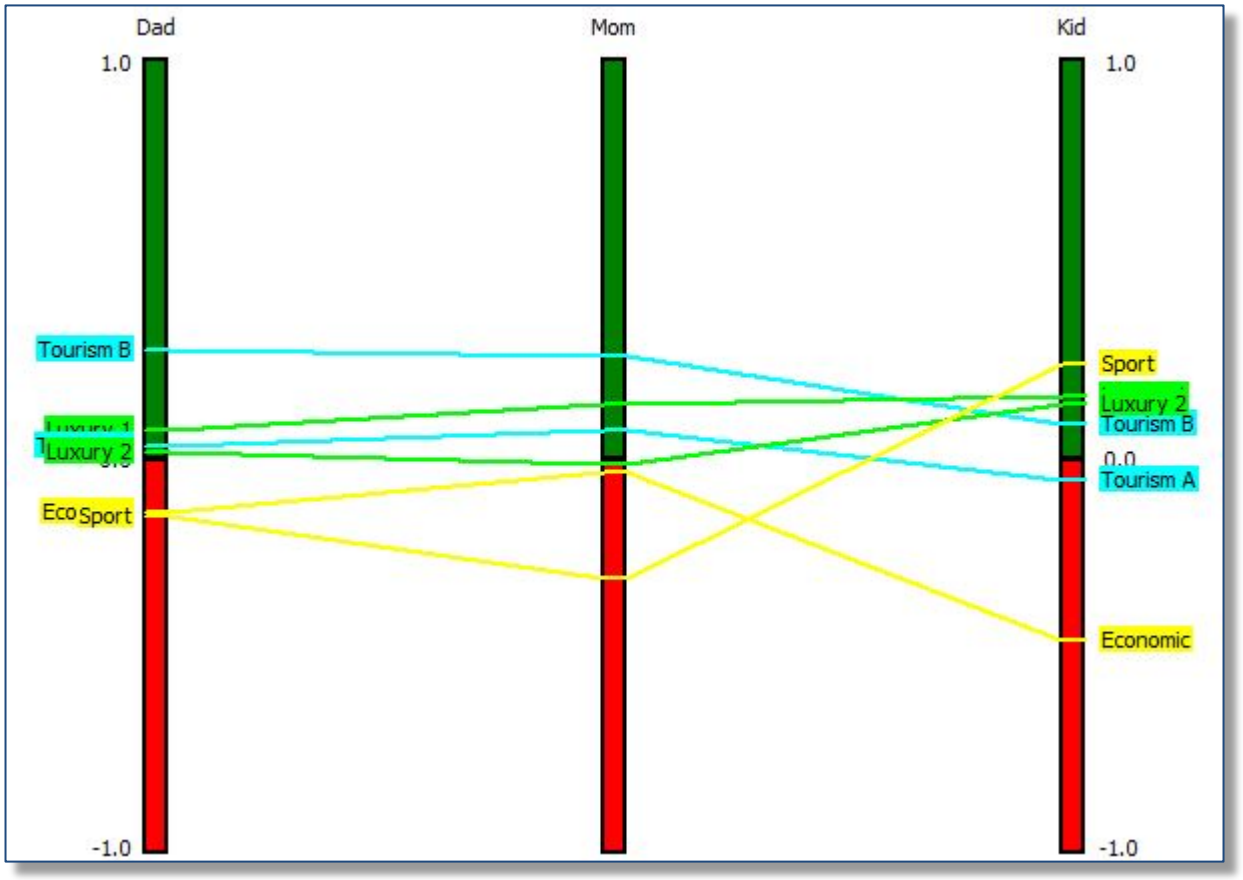
Multi-scenarios Model

- Scenarios:
  - Points of view (stakeholders – GDSS)
  - Hypotheses, …
- Evaluations:
  - ‘Objective’ criteria: common evaluations.
  - ‘Subjective’ criteria: specific evaluations for each scenario.
- Specific preference structures :
  - Weights, preference thresholds.

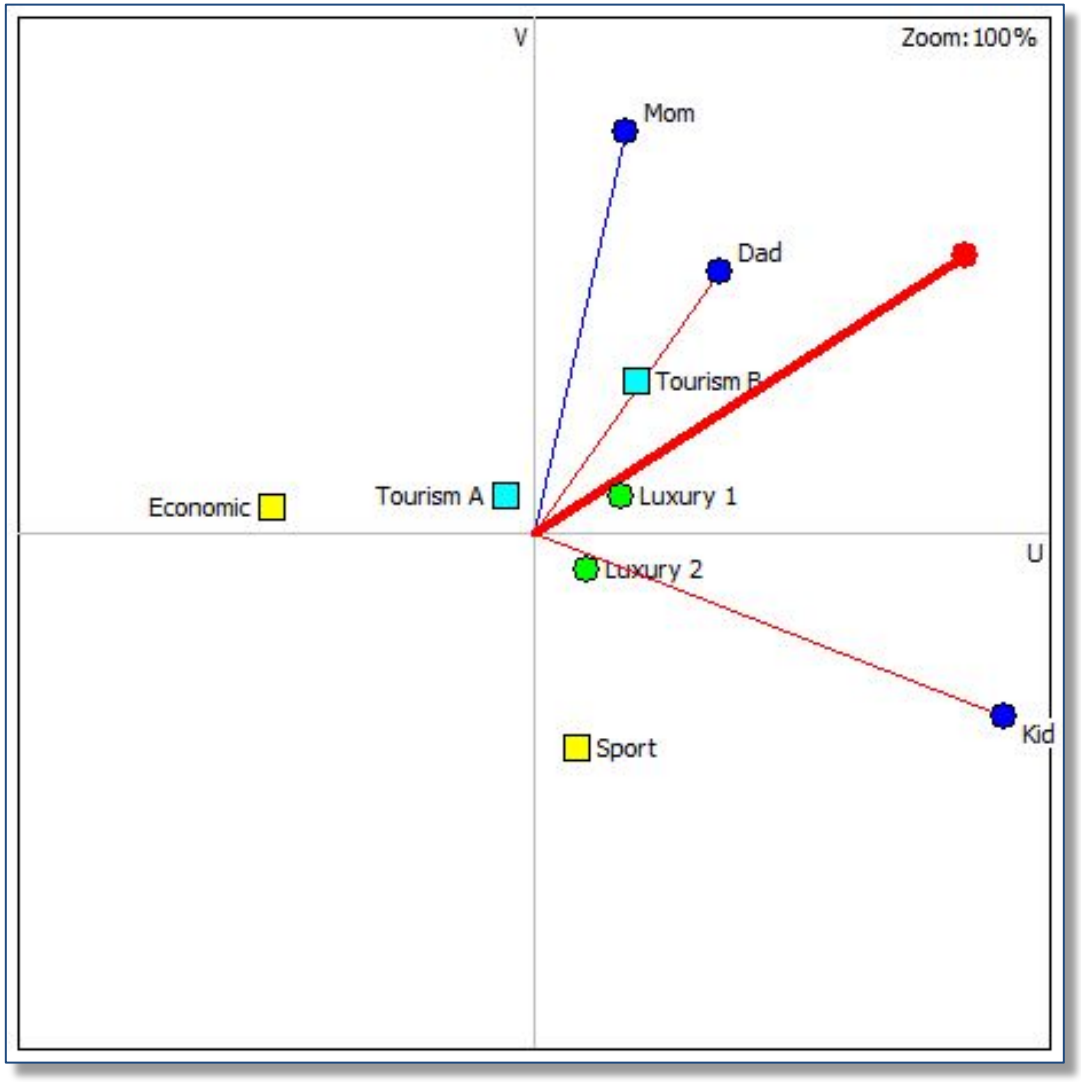
Multi-scenarios Model

- Adaptation of **PROMETHEE**:
  - Individual rankings,
  - Global (group) ranking with possible weighing of the scenarios.
- Adaptation of **GAIA**:
  - Three different analyses:
    - **GAIA**-Criteria,
    - **GAIA**-Scenarios,
    - **GAIA**-Unicriterion.

Individual **PROMETHEE** rankings

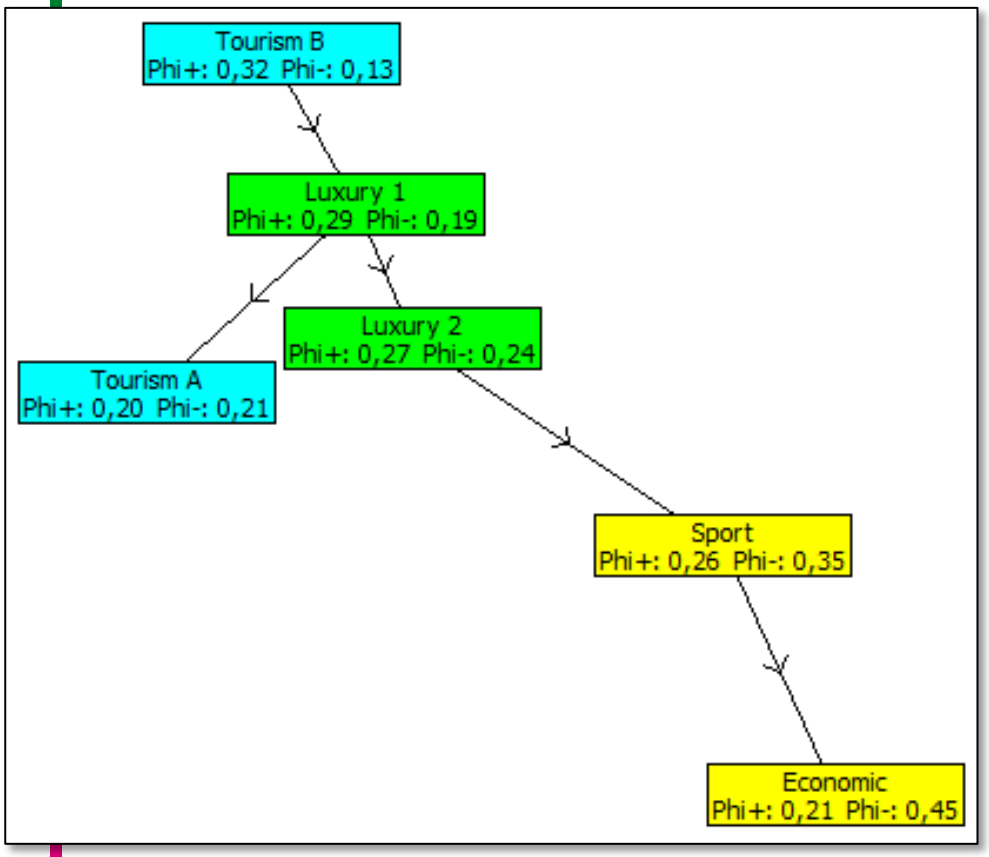


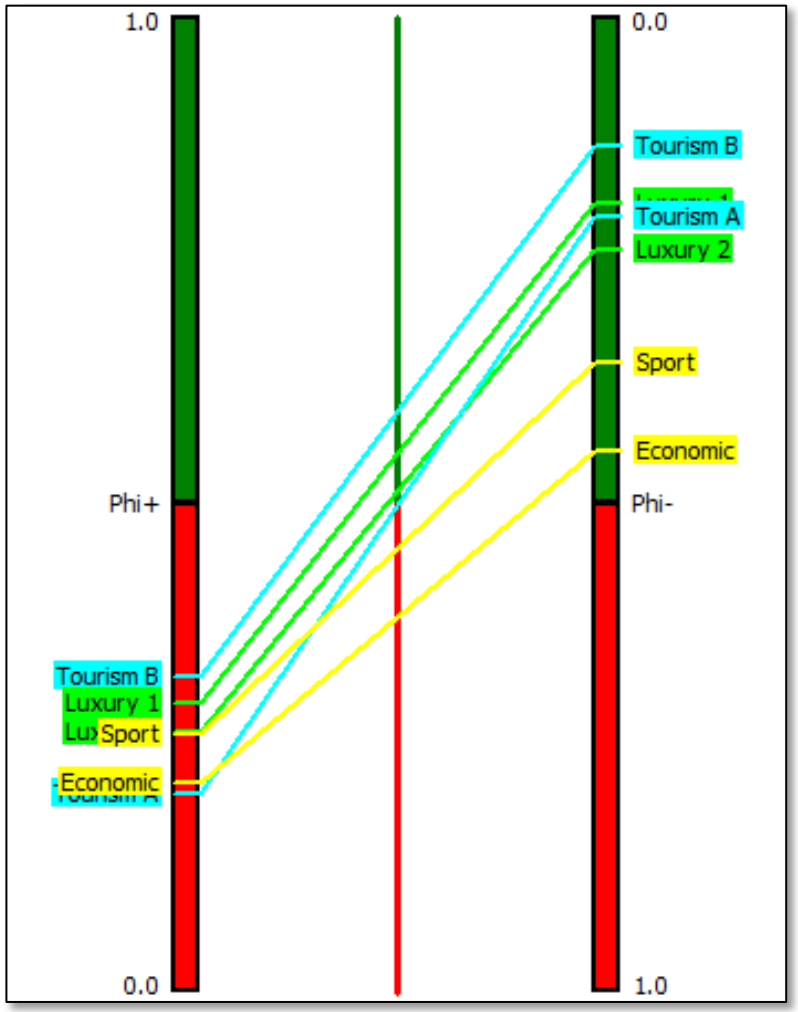
GDSS-**GAIA**: Scenarios



GDSS-**GAIA**: Criterion Comfort

« Family » (group) ranking





Visual PROMETHEE

WWW.PROMETHEE-GAIA.NET

- **Visual PROMETHEE** software:
  - **Free Academic Edition**
  - **Business Edition**
- **Visual PROMETHEE** Manual (PDF or ebook)
- More information:
  - Documents available at www.promethee-gaia.net
  - biblio.promethee-gaia.net :
    - Over 2000 references available.
  - Services: Training, Coaching, Free seminars
  - LinkedIn group, Twitter, ResearchGate, …

What next?

- First: Thanks to all of you! 谢谢
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