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 <small>UNIVERSITÉ LIBRE DE BRUXELLES</small> 	 <h1>MultiCriteria Decision Aid with <b>PROMETHEE &amp; GAIA</b></h1> <p><i>How to make better and more sustainable decisions</i></p> <p><b>Πανεπιστήμιο Μακεδονίας</b> Θεσσαλονίκη, Ιουλίου 4-6, 2017</p> <p>Bertrand MARESCHAL Solvay Brussels School of Economics &amp; Management <a href="mailto:bmaresc@ulb.ac.be">bmaresc@ulb.ac.be</a> <a href="http://www.promethee-gaia.net">http://www.promethee-gaia.net</a></p>
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## Course Schedule



- **Tuesday 4 – What? Why? How?**
  - What is multicriteria decision aid.
  - Why you should use multicriteria decision aid.
  - **PROMETHEE & GAIA**
- **Wednesday 5 – The software**
  - Advanced features.
  - Using **PROMETHEE** and the **Visual PROMETHEE** software.
  - Hands-on training.
- **Thursday 6 – Practice**
  - Exercises.
  - Case studies (homeworks).

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## **Tuesday 4**

### What? Why? How?



- What is multicriteria decision aid:
  - Optimization vs multicriteria decision aid.
  - Sustainable decisions.
  - Models and main approaches.
- Why you should use multicriteria decision aid.
- **PROMETHEE & GAIA.**

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## Decisions



- Personal decisions
  - Choose a restaurant tonight
  - Choose a university
  - Purchase a new phone, a new car, ...
- Business decisions
  - Develop a new product
  - Choose a computer system
  - Investments, strategies, project management, ...
- Political decisions
  - Join the EU... Leave the EU...
  - Build a new hospital
  - Regional investment, taxes, ...





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## Decision Making




- Describe,
- Understand,
- Manage.

**2 Approaches :**

- Qualitative approach,
- Quantitative approach.

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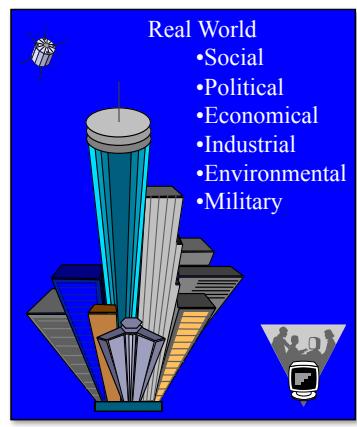
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## Decision Aid



Real World

- Social
- Political
- Economical
- Industrial
- Environmental
- Military



Quantitative model

- Possible decisions?
- How to compare them?
- Preferences, Objectives?

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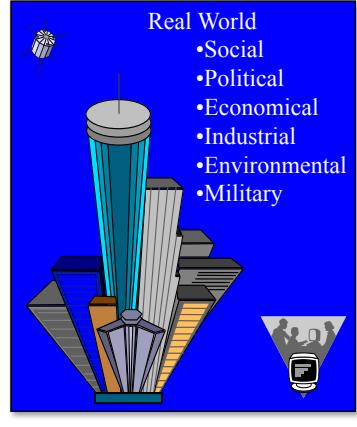
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## Decision Aid



Real World

- Social
- Political
- Economical
- Industrial
- Environmental
- Military



Quantitative model

- Approximation to real world!
- Decision Aid.

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# Quantitative Model?

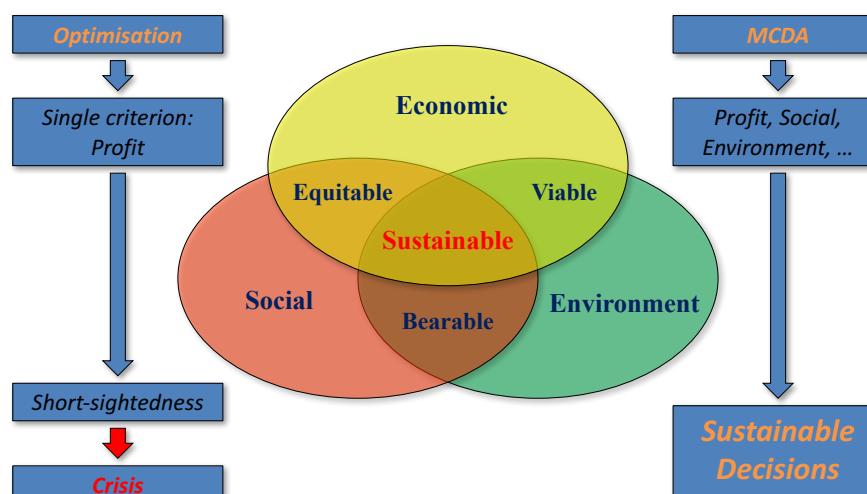


- Describe possible decisions (actions)?
  - List, variables, ...
- Objective? Best decision? Best choice?
  - Minimize costs?
  - Maximize profit?
  - Maximize quality?
  - Minimize impacts?
- Optimization models?
- Or MCDA (MultiCriteria Decision Aid) models...

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# MCDA vs Optimisation



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## Some Decision or Evaluation Problems



- Locating a new plant, a new shop, ...
- Human resources management.
- Purchasing equipment.
- Assessing the quality of suppliers.
- Evaluating projects.
- Selecting an investment strategy.
- Making political decisions...

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## What? Why? How?



- What are we facing?
  - Decision problems involving multiple and conflicting issues.
  - Decision problems involving the future of Mankind.
- Why should we care?
  - Robin Hood is dead.
  - Short-term profit is ruling.
- How can we face it?
  - Think ethical.
  - Share. React. Act.
  - Need for models to avoid subjectivity and unethical behavior.



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## Which models?



- Qualitative models?
  - Subjective!
- Unicriterion models?
  - Less subjective,
  - Short-sighted!
- Multicriteria models?
  - More open,
  - Still models...

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## Unicriterion vs Multicriteria Model



- **Unicriterion model:**

$$\text{Optimise} \{g(a) | a \in A\}$$
  - Mathematically **well-stated**:
    - Optimal solution,
    - Complete ranking of the actions.
  - Socio-economically **ill-stated**:
    - Single criterion? Not realistic.
    - Notion of criterion: perception thresholds, ...

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## Unicriterion vs Multicriteria Model



- **Multicriteria model:**

$$\text{Optimise} \{g_1(a), g_2(a), \dots, g_k(a) | a \in A\}$$

- Mathematically **ill-stated**:

- No optimal solution,
- No mathematical meaning.

- Socio-economically **well-stated**:

- Closer to real world decision problem,
- Search for a compromise solution.

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## Multicriteria Table



- Actions:
  - Possible decisions,
  - items to evaluate.
- Criteria:
  - quantitative,
  - qualitative.

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## Multicriteria Table



	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	...
Action 1	18	135	G	Yes	...
Action 2	9	147	B	Yes	...
Action 3	15	129	VG	No	...
Action 4	12	146	VB	?	...
Action 5	7	121	G	Yes	...
...	...	...	...	...	...

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## Plant Location



	Investment (M€)	Costs (k€)	Environm. (impact)	...
Site 1	18	135	G	...
Site 2	9	147	B	...
Site 3	15	129	VG	...
Site 4	12	146	VB	...
Site 5	7	121	G	...
...	...	...	...	...

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## Purchase Options



	Price (€)	Reliability (days)	Maintenance (estimate)	...
Product A	18	135	G	...
Product B	9	147	B	...
Product C	15	129	VG	...
Product D	12	146	VB	...
Product E	7	121	G	...
...	...	...	...	...

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## A Simple Example



*The purchase of a new car  
by a Greek family*

**Objectives:**

- Economy (price),
- Usage (fuel consumption),
- Performance (power),
- Space,
- Comfort.

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## Multicriteria Table



Cars	Price	Power	Fuel	Space	Comfort
Tourism A	26.000 €	75	8,0	average	average
Sport	29.000 €	110	9,0	very bad	bad
Tourism B	25.500 €	85	7,0	good	average
Luxury 1	38.000 €	90	8,5	good	very good
Economic	15.000 €	50	7,5	bad	very bad
Luxury 2	35.000 €	85	9,0	very good	good

- Best buy?
- Best compromise?
- Priorities of the buyer?



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## Who is the decision maker?

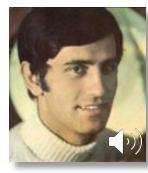




**Dad (Ιάσων)?**  
He would prefer to save money  
for holidays in sunny Belgium but  
needs space for mom's suitcases...



**Mom (Ελευθερία)?**  
She would prefer a red one...



**The kid (Γιώργος)?**  
He doesn't care too much about  
money... More about speed and  
ρεμπέτικο τραγούδι!

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## One or Several Stakeholders ?



- Single stakeholder:
  - One actor (the decision maker).
  - One multicriteria table and one preference structure.
- Multiple stakeholders:
  - Several actors (including decision maker(s)).
  - Several multicriteria tables and preference structures.
  - Search for consensus.

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## Example



- A Greek family.
- Three stakeholders (“decision makers”):
  - Ιάσων (dad),
  - Ελευθερία (mom),
  - Γιώργος (the kid).
- Three scenarios.
- Three multicriteria tables:
  - Different priorities.
  - Subjective evaluation of comfort.

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## Homework Assignment



- For Thursday.
- By small groups of 4 to 6 students.
- Set up a multicriteria decision problem:
  - Actual or fictive.
  - Including minimum 6 actions, 5 criteria and 2 scenarios.
  - Prepare the multicriteria evaluation table.
  - Think about preference functions and criteria weights.

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## How to?

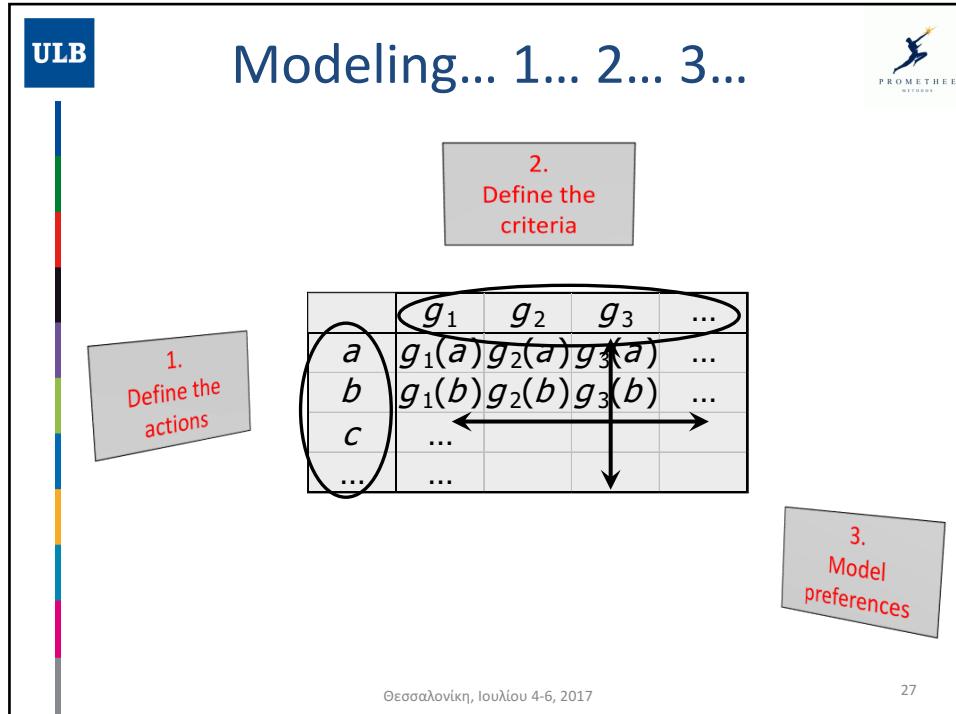


- What is the problem?
- What are the issues?
- Who is involved (stakeholders)?
- What is possible (decisions)?
- What is at stake (objectives)?
- How to measure achievement (criteria)?

➤ Modeling...

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## 1. Defining the actions

PROMETHEE METHODS

- Challenge:
  - Be exhaustive.
  - Open the options.
- Definition: Let  $A$  the set of actions.  $A$  can be defined:
  - **in extension:**  
by enumeration of its elements.  
→ relatively small number of actions.
  - **in comprehension:**  
by constraints on a set of decision variables.  
(Cf. linear programming)  
→ large number or infinity of actions.

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## Some properties of the set of actions



The set of actions  $A$  can be:

- **stable**: a priori defined, doesn't evolve.  
or
- **evolutive**: can evolve during the procedure.
- **globalized**: mutually exclusive elements.  
or
- **fragmented**: combinations of actions are considered.

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## 2. Defining the criteria



- **Definition:**  
function  $g$  defined on  $A$ , taking its values in a totally ordered set, and representing an objective of the decision-maker(s).
- **Challenge:**
  - Consistent family of criteria:
    - Include all aspects of the decision problem, all the objectives of the decision-maker(s),
    - Avoid redundancies.

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## Qualitative vs Quantitative Criteria



- Quantitative criteria:
  - Natural numerical scale.
- Qualitative criteria:
  - Qualitative ordinal scale (ex: Likert scales).
  - Maximum 9 levels ( $7 \pm 2$ ) to ensure a consistent evaluation.
  - Presence of a neutral level?
  - Examples:
    - Very good, Good, Average, Bad, Very bad
    - Yes, No
    - ++, +, 0, -, --
    - ++, +, -, --
  - Underlying numerical scale (coding).

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## 3. Modeling preferences



- Problem:  
 How to compare two actions  
 $a$  and  $b$  to each other?
- A first model: 3 possible results:
  1. Preference:  $aPb$  or  $bPa$
  2. Indifference:  $aIb$
  3. Incomparability:  $aRb$

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## Traditional preference structure (unicriterion)



- Optimisation of a function  $g$  on  $A$

$$\forall a, b \in A : \begin{cases} aPb & \Leftrightarrow g(a) > g(b) \\ aIb & \Leftrightarrow g(a) = g(b) \end{cases}$$

- Consequences:

$R$ is empty
$P$ is transitive
$I$ is transitive

- Complete ranking.

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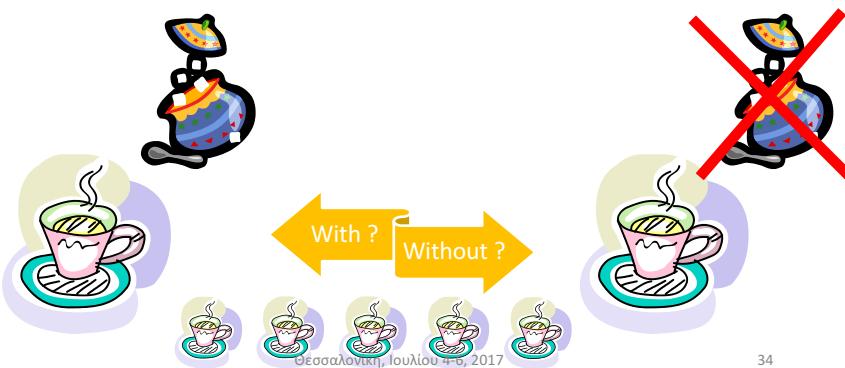
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## The notion of indifference threshold



- Problem: Indifference can be intransitive.

Cf. Coffee cup paradox (Luce, 1956)



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## The notion of indifference threshold



- Problem: Indifference can be intransitive.  
Cf. Coffee cup paradox (Luce, 1956)
  - Introduction of an indifference threshold:
- $$\forall a, b \in A : \begin{cases} aPb & \Leftrightarrow g(a) > g(b) + q \\ aIb & \Leftrightarrow |g(a) - g(b)| \leq q \end{cases}$$
- Quasi-order :  $P$  is transitive, but not  $I$ .

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## Other preference structures



- Variable indifference threshold  
⇒ Interval order.
- Preference + indifference thresholds  
⇒ Pseudo-order.
- Models including incomparability  
⇒ Partial orders.
- Valued preference structures.

Used by  
PROMETHEE

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## Different Problematics



	$g_1$	$g_2$	$g_3$	...
$a$	$g_1(a)$	$g_2(a)$	$g_3(a)$	...
$b$	$g_1(b)$	$g_2(b)$	$g_3(b)$	...
$c$	...			
...	...			

**Evaluations**

- $n$  actions
- $k$  criteria

- $\alpha$  - **choice**: determine a subset of actions (the « best ones »).
- $\beta$  - **sorting**: sort actions in predefined categories.
- $\gamma$  - **ranking**: rank from the best to the worst action.
- $\delta$  - **description**: describe actions and their consequences.

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## Unicriterion Model: Optimization



- Optimal solution
  - Maximum or minimum of criterion value.
  - Mathematically well-defined.
  - Generally exists.
- Optimization algorithms
  - Adapted to specific cases (linear programming, branch and bound, network optimization, ...).
  - “Proved”.

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# Multicriteria Model: Dominance and efficiency



- « Objective ».
  - Based on a unanimity principle:
- $$a \text{ dominates } b \Leftrightarrow g_h(a) \geq g_h(b) \quad \forall h$$
- Efficiency:  $a$  is efficient if it is not dominated by any other action.
  - Problems:
    - Dominance is poor (few dominances),
    - Many actions are efficient.

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# Multicriteria Table



Cars	Price	Power	Fuel	Space	Comfort
Tourism A	26.000 €	75	8,0	average	average
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- Best buy?
- Best compromise?
- Priorities of the buyer?

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## Objections to Dominance



I	$g_1$	$g_2$
$a$	100	100
$b$	20	30

- $a$  efficient
- $a$  preferred to  $b$

II	$g_1$	$g_2$
$a$	100	30
$b$	20	100

- $a$  and  $b$  efficient
- $a$  and  $b$  incomp.

III	$g_1$	$g_2$
$a$	100	99
$b$	20	100

- $a$  and  $b$  efficient
- $a$  preferred to  $b$

IV	$g_1$	$g_2$
$a$	100	99
$b$	99	100

- $a$  and  $b$  efficient
- $a$  and  $b$  indifferent

V	$g_1$	$g_2$
$a$	100	100
$b$	99	99

- $a$  efficient
- $a$  and  $b$  indifferent

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## Some Characteristics for a good multicriteria method



- Take into account **deviations** between evaluations.
- Take **scale** effects into account.
- Build either a **partial ( $P,I,R$ )** or a **complete ( $P,I$ )** ranking of the actions.
- Stay sufficiently **simple**:
  - **no** black box,
  - **no** technical parameters.

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## A common approach: The weighted sum

	$g_1$	$g_2$	$g_3$	...
$a$	$g_1(a)$	$g_2(a)$	$g_3(a)$	...
$b$	$g_1(b)$	$g_2(b)$	$g_3(b)$	...
$c$	...	...	...	...
...	...	...	...	...
	$w_1$	$w_2$	$w_3$	...

Actions or Decisions

Weights of the criteria

Criteria

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## A common approach: The weighted sum

- Global value for  $a$  :

$$V(a) = w_1 g_1(a) + w_2 g_2(a) + \dots$$

- $a$  is preferred to  $b$  if:

$$V(a) > V(b)$$

(if all criteria are to maximise)

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## Weighted Sum



### Example 1

	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$
$a$	100	100	100	100	55
$b$	85	85	85	85	100
	1/5	1/5	1/5	1/5	1/5

- $V(a) = 91 \quad V(b) = 88$
- Total and uncontrolled compensation of weaknesses by strengths.

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## Weighted Sum



### Example 2

	$g_1$	$g_2$
$a$	100	0
$b$	0	100
$c$	50	50
$d$	50	50
	1/2	1/2

- $V(a) = V(b) = V(c) = V(d) = 50$
- Elimination of conflicts – Loss of information.

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## Weighted Sum

### Example 3



*“Profit is approximately 2 times more important than time savings;  
0.7 for profit and 0.3 for time savings.”*

	$g_1$ (BF)	$g_2$ (min)
$a$	60	60
$b$	48	70
	0.7	0.3

$$\begin{aligned} V(a) &= 60 \\ V(b) &= 54.6 \\ a \text{ is ranked } 1^{\text{st}}. \end{aligned}$$

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## Weighted Sum

### Example 3



*“Profit is approximately 2 times more important than time savings;  
0.7 for profit and 0.3 for time savings.”*

	$g_1$ (FF)	$g_2$ (min)
$a$	10	60
$b$	8	70
	0.7	0.3

$$\begin{aligned} V(a) &= 25 \\ V(b) &= 26.6 \\ b \text{ is ranked } 1^{\text{st}}! \end{aligned}$$

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## Weighted Sum

### Example 3

	$g_1$ (BF)	$g_2$ (min)
$a$	60	60
$b$	48	70
	0.7	0.3

$V(a) = 60$   
 $V(b) = 54.6$

$a$  is ranked 1st.

	$g_1$ (FF)	$g_2$ (min)
$a$	10	60
$b$	8	70
	0.7	0.3

$V(a) = 25$   
 $V(b) = 26.6$

$b$  is ranked 1st.

$\rightarrow$  Significance of the “weights” !  $\leftarrow$

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## Multicriteria Decision Aid



- Multiattribute utility theory (MAUT).
- Outranking methods.
- Interactive methods.
- Multiobjective programming.
- ...

Since 1970, numerous developments:  
 conferences, papers, books,  
 applications, software...

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## Multiattribute Utility Theory



- Single synthesis criterion (aggregation).

$$U(a) = U(g_1(a), g_2(a), \dots, g_k(a))$$

- Existence?
- Construction?
- Mathematical form?

→ additive?

$$U(a) = \sum_{j=1}^k U_j(g_j(a))$$

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## Multiattribute Utility Theory



- Construction method:
  - direct,
  - indirect.
- Information-intensive for the decision maker.  
(quantity of information vs reliability?).
- Not flexible (sensitivity analyses).
- Far away from the original decision problem structure:

multicriteria → unicriterion

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## Outranking Methods



- Majority principle  
(vs unanimity for dominance).
- Pairwise comparison of actions.
- Closer to the decision problem.
- ELECTRE methods (1968-).
- **PROMETHEE & GAIA** methods (1982-).

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## Tuesday 4



### What? Why? How?

- What is multicriteria decision aid.
- Why you should use multicriteria decision aid:
  - Facing complexity and conflicting issues.
  - Single decision maker context.
  - Collaborative group decision.
  - Negotiations between parties or stakeholders.
- **PROMETHEE & GAIA.**

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## Tuesday 4

# What? Why? How?

- What is multicriteria decision aid.
- Why you should use multicriteria decision aid.
- **PROMETHEE & GAIA:**
  - Data and preference modeling.
  - **PROMETHEE** rankings.
  - **GAIA** visual analysis.
  - Sensitivity analyses.
  - Group decision and negotiation.

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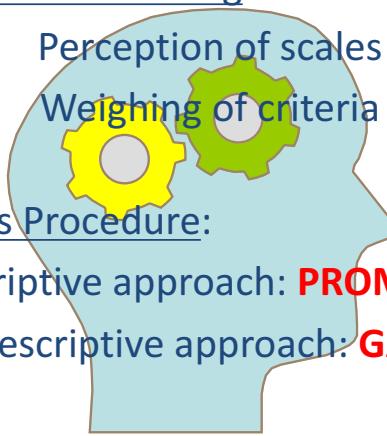
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## Decision Aid Methods

- Preference modelling:

Perception of scales
Weighing of criteria
Analysis Procedure

- Analysis Procedure:
  - Prescriptive approach: **PROMETHEE**
  - Descriptive approach: **GAIA**

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## Why PROMETHEE?



- Proven methodology:
  - 30 years development,
  - Over 1570 published scientific papers.
- « Simplicity ».
- Visual tools.
- Sensitivity analysis tools.
- Interactivity.
- **Visual PROMETHEE software.**

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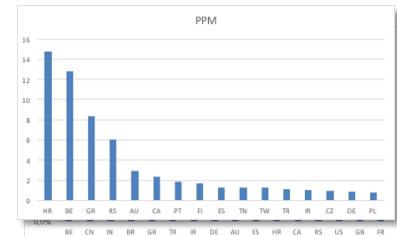
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## Some statistics...



- First paper published in **1982** by J-P. Brans.
- Over **1570** published papers as of today.
- **92** papers published by **153** Greek authors, from **1989** to 2017  
(worldwide #**5**, after Belgium, China, India and Brazil; #**3** PPM)
- Main fields of application:
  - Environment
  - Industry
  - Services / Public sector
  - Energy
  - Finance



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**PROMETHEE Timeline**

- Over 1570 papers published:
  - 75% applied – 25% theoretical
  - 55% related to “societal” fields
- Median year: 2011
- Over 2700 authors from 79 countries.

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**Principles of the PROMETHEE Methods**

- Preference modelling:
  - Preference functions,
  - Weighing of the criteria.
- Pairwise comparison of the actions:
  - Outranking,
  - Prudent (partial ranking),
  - Partially compensatory approach:
    - Advantage over weighted sum and utility functions.

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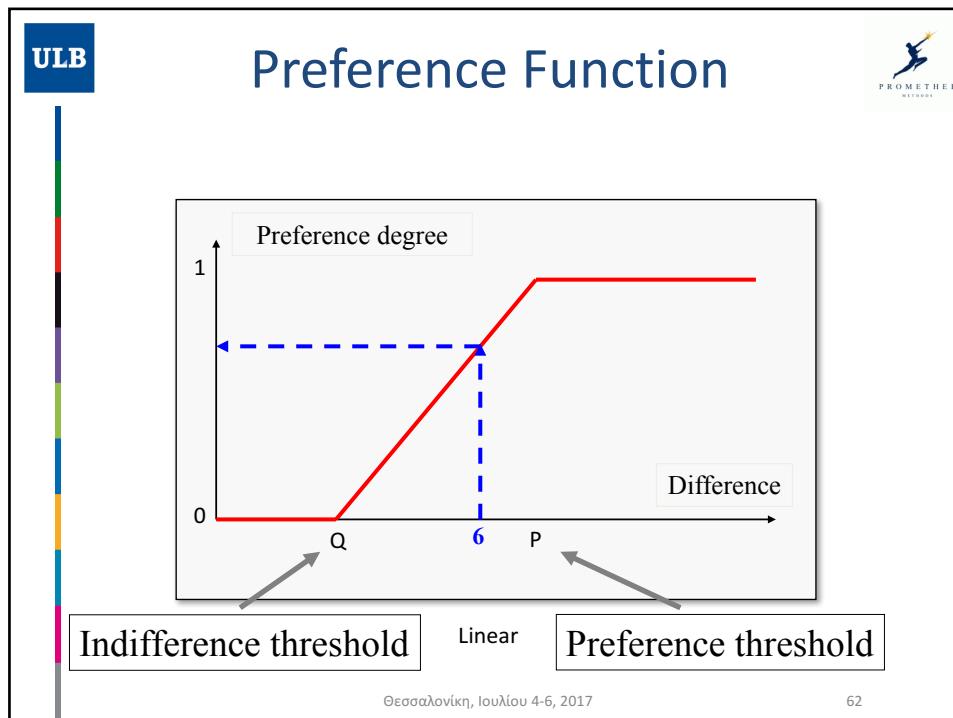
## Comparison of 2 Actions

PROMETHEE METHODS

	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	...
Action 1	18	135	G	Yes	...
Action 2	9	147	Difference = 6		...
Action 3	15	129	VG	No	...
Action 4	12	146	VB	?	...
Action 5	7	121	G	Yes	...
...	...	...	...	...	...

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# PROMETHEE

**Pref (Eco.,Lux.)**

	Economic	Luxury 1	Wght
1,0	15000	38000	0,0 1/5
0,0	50	Power 90	1,0 1/5
0,5	-1,0	Fuel 8,5	0,0 1/5
0,0	7,5	Space G	+2 0,5 1/5
0,0	B	Comfort VG	+4 1,0 1/5
	VB		

**Pref (Lux.,Eco.)**

	Economic	Luxury 1	Wght
1,0	15000	38000	0,0 1/5
0,0	50	Power 90	1,0 1/5
0,5	-1,0	Fuel 8,5	0,0 1/5
0,0	7,5	Space G	+2 0,5 1/5
0,0	B	Comfort VG	+4 1,0 1/5
	VB		

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$\text{Pref (Eco.,Lux.)} = 0,3$   
 $= (1 + 0 + 0,5 + 0 + 0) / 5$

$\text{Pref (Lux.,Eco.)} = 0,5$   
 $= (0 + 1 + 0 + 0,5 + 1) / 5$

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# PROMETHEE

**Pref (Eco.,Lux.)**

	Economic	Luxury 1	Wght
1,0	15000	38000	0,0 2/7
0,0	50	Power 90	1,0 1/7
0,5	-1,0	Fuel 8,5	0,0 2/7
0,0	7,5	Space G	+2 0,5 1/7
0,0	B	Comfort VG	+4 1,0 1/7
	VB		

**Pref (Lux.,Eco.)**

	Economic	Luxury 1	Wght
1,0	15000	38000	0,0 2/7
0,0	50	Power 90	1,0 1/7
0,5	-1,0	Fuel 8,5	0,0 2/7
0,0	7,5	Space G	+2 0,5 1/7
0,0	B	Comfort VG	+4 1,0 1/7
	VB		

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$\text{Pref (Eco.,Lux.)} = 0,43$   
 $= (2 \times 1 + 0 + 2 \times 0,5 + 0 + 0) / 7$

$\text{Pref (Lux.,Eco.)} = 0,36$   
 $= (0 + 1 + 0 + 0,5 + 1) / 7$

## Pairwise Comparisons



- For each criterion  $g_j$  :
  - Preference function  $P_j$
  - Weight  $w_j$
- Multicriteria preference degree of  $a$  over  $b$  :

$$\pi(a,b) = \sum_{j=1}^k w_j P_j(a,b)$$

– Properties:

$$0 \leq \pi(a,b) \leq 1$$

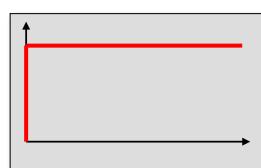
$$\pi(a,b) > 0 \Rightarrow \pi(b,a) = 0$$

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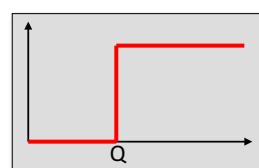
65

## Preference Functions

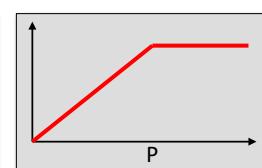
(as in **Visual PROMETHEE** software)



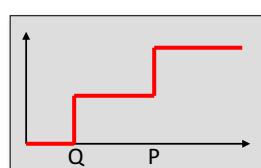
Usual



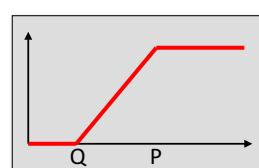
U-shape



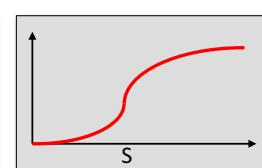
V-shape



Level



Linear



Gaussian

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# PROMETHEE

PROMETHEE METHODS

		Pref (Eco.,Lux.)		Pref (Lux.,Eco.)		
		Economic		Luxury 1		
1,0	-23000	15000	Price	38000	0,0	
0,0	-1,0	50	Power	90	1,0	
0,5		7,5	Fuel	8,5	0,0	
0,0		B	Space	G	0,5	
0,0		VB	Comfort	VG	+4	1,0

Preference

0 q p Deviation

Economic

Luxe 1

Pairwise comparisons

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## Computation of Preference Flows

PROMETHEE METHODS

$\pi(a,b)$	Tour.A	Sport	Tour.B	Lux.1	Econ.	Lux.2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
Lux.1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
Lux.2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$	0,02	-0,17	0,26	0,06	-0,15	0,00	

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## Preference Flows



- Relative scores computed for the actions.
- Leaving (+) and entering (-) flows:
  - Strength:  $0 \leq \phi^+ \leq 1$
  - Weakness:  $0 \leq \phi^- \leq 1$
- Net flow:
  - Balance:  $-1 \leq \phi = \phi^+ - \phi^- \leq +1$
- Unicriterion net flows:
  - Standardized scores for each criterion:  
 $\text{criterion } f_j \Rightarrow -1 \leq \phi_j \leq +1$

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## Wednesday 5

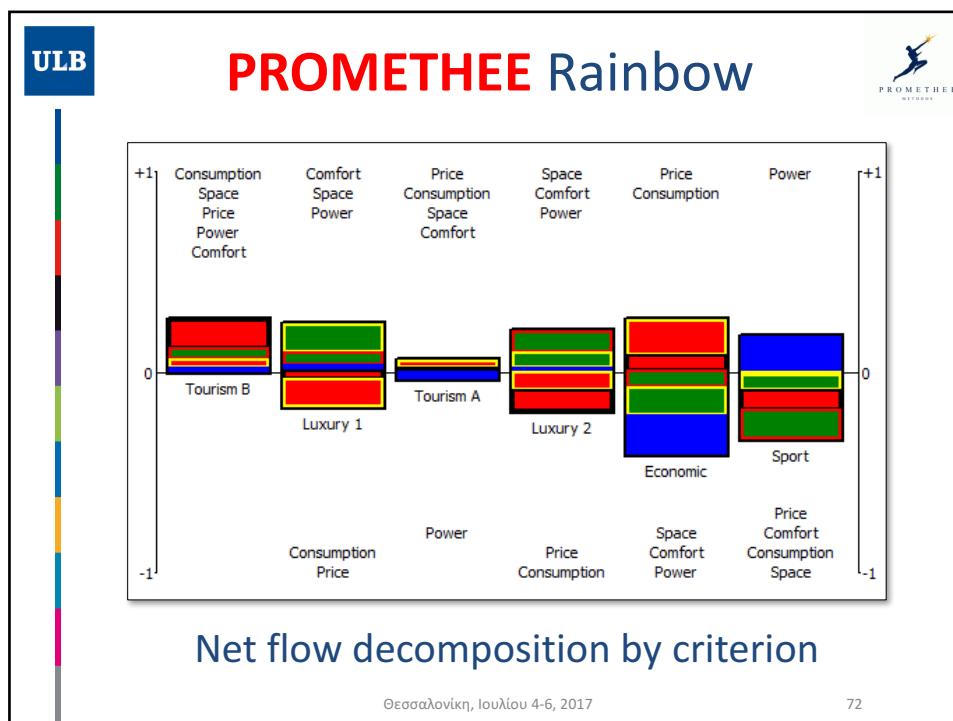
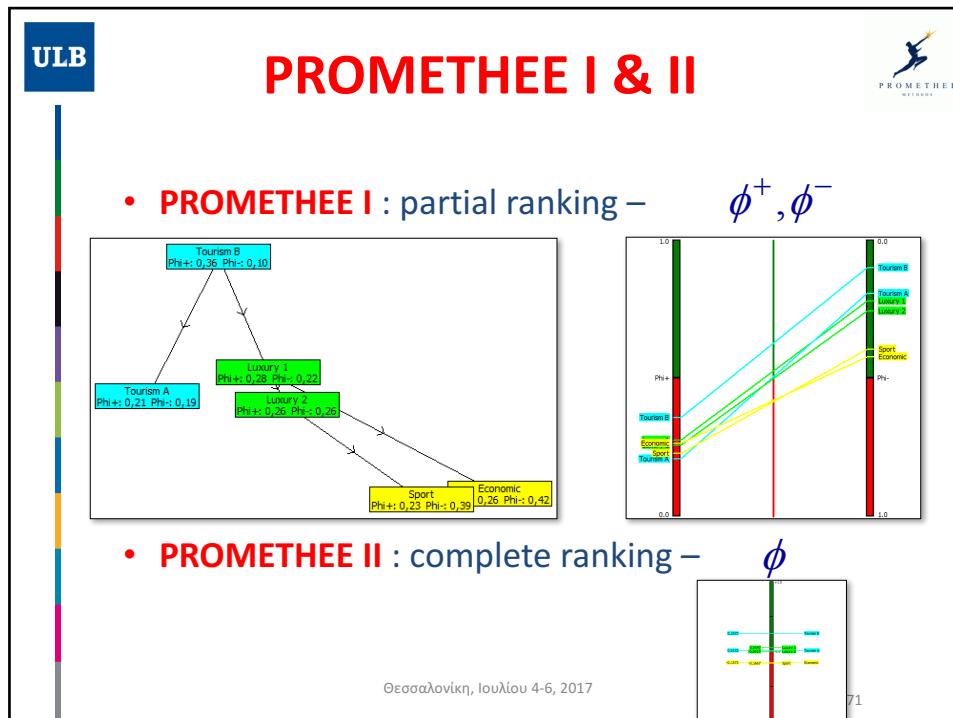


### The Software

- Advanced features.
- Using PROMETHEE and the Visual PROMETHEE software.
- Hands-on training.

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# Sensitivity Analysis with PROMETHEE

PROMETHEE METHODS

- Criteria weights ↔ PROMETHEE ranking.
- Interactive weight sensitivity analysis: « Walking Weights ».
- Robustness with respect to weight values?
  - Weight stability intervals.
  - Visual weight stability intervals.

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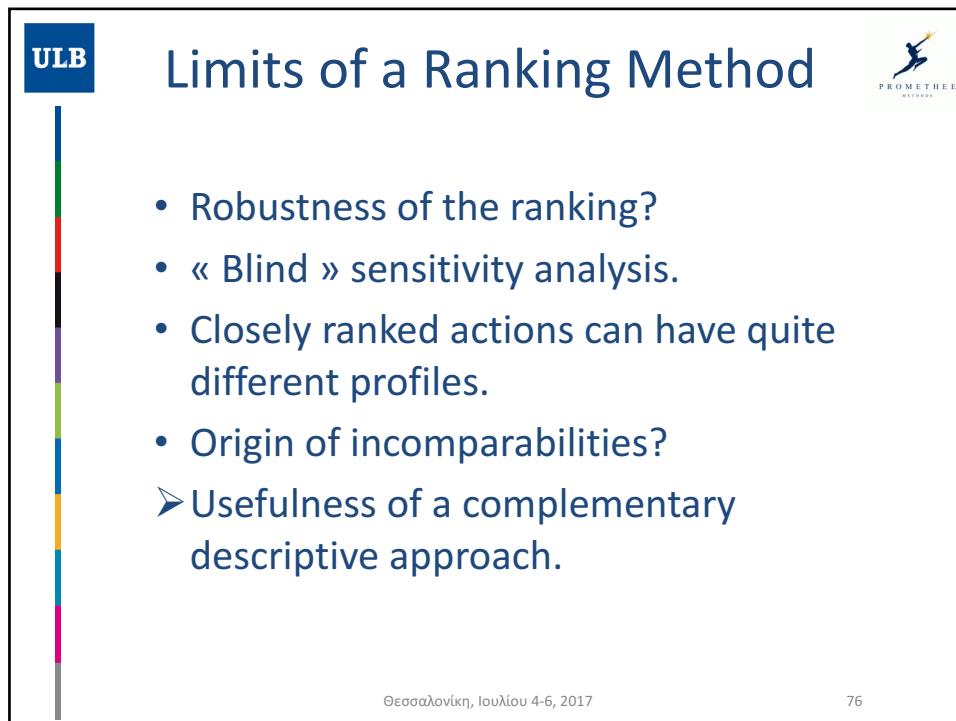
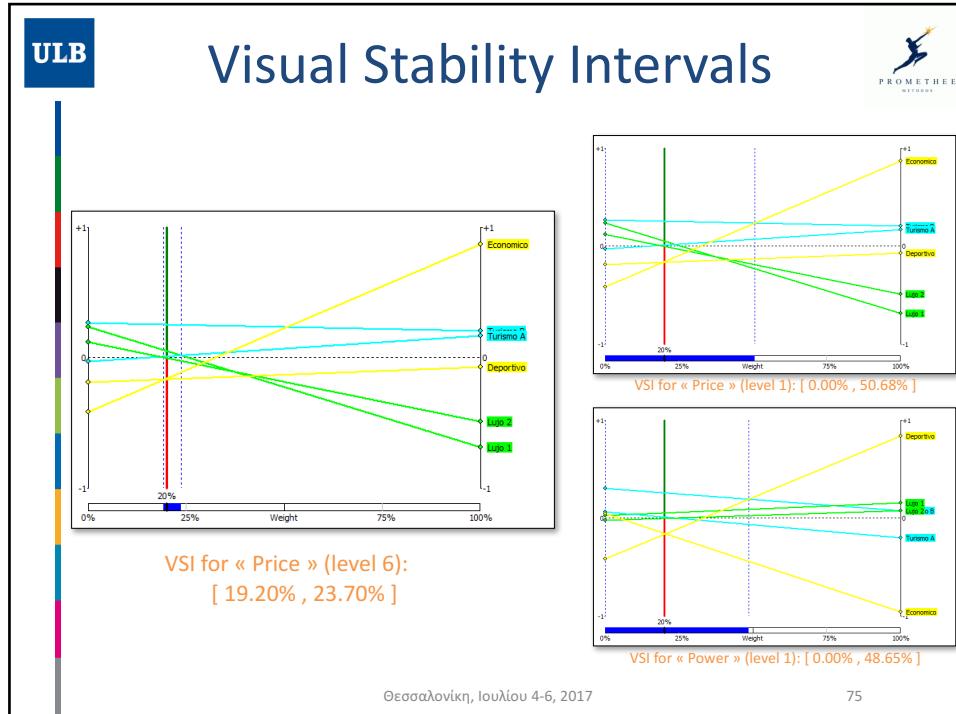
## Walking Weights

PROMETHEE METHODS

The figure illustrates the concept of "Walking Weights". It shows two types of bar charts. On the left, a standard horizontal bar chart displays values for categories like Tourism B, Luxury 1, Tourism A, Luxury 2, Economic, and Sport. On the right, a "Walking Weights" chart shows the same categories but with their bars shifted to the right, creating a visual representation of how changes in criteria weights affect the ranking. Arrows indicate the transformation between the two types of charts.

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## Properties of the Net Flow



- Net flow is centered:  $\sum_{a \in A} \phi(a) = 0$

- Unicriterion net flows:

$$\phi(a) = \sum_{j=1}^k w_j \cdot \phi_j(a)$$

with

$$\phi_j(a) = \frac{1}{n-1} \sum_{b \in A} [P_j(a, b) - P_j(b, a)]$$

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## GAIA



- Visual descriptive analysis.
- Better understanding:
  - Conflicting criteria.
  - Action profiles.
  - Possible compromise solutions.
- Reducing the multicriteria dimension:
  - Principal components analysis.

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# GAIA

PROMETHEE METHODS

1. Computation of unicriterion net flows (normalization)

2. Projection on a plane:

- Graphical representation.
- 5 dimensions!

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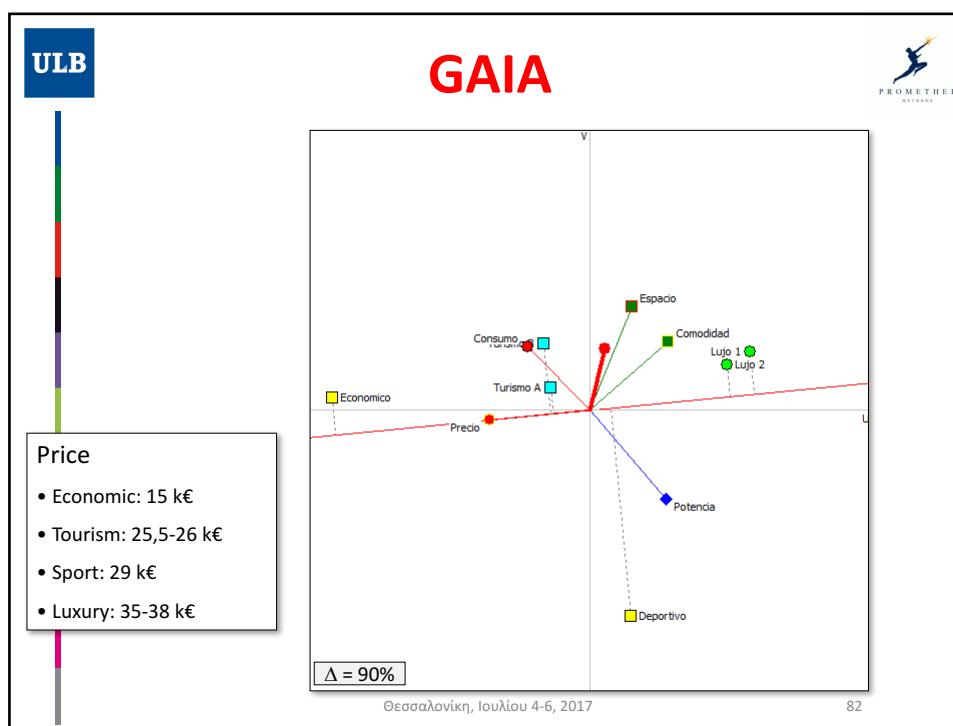
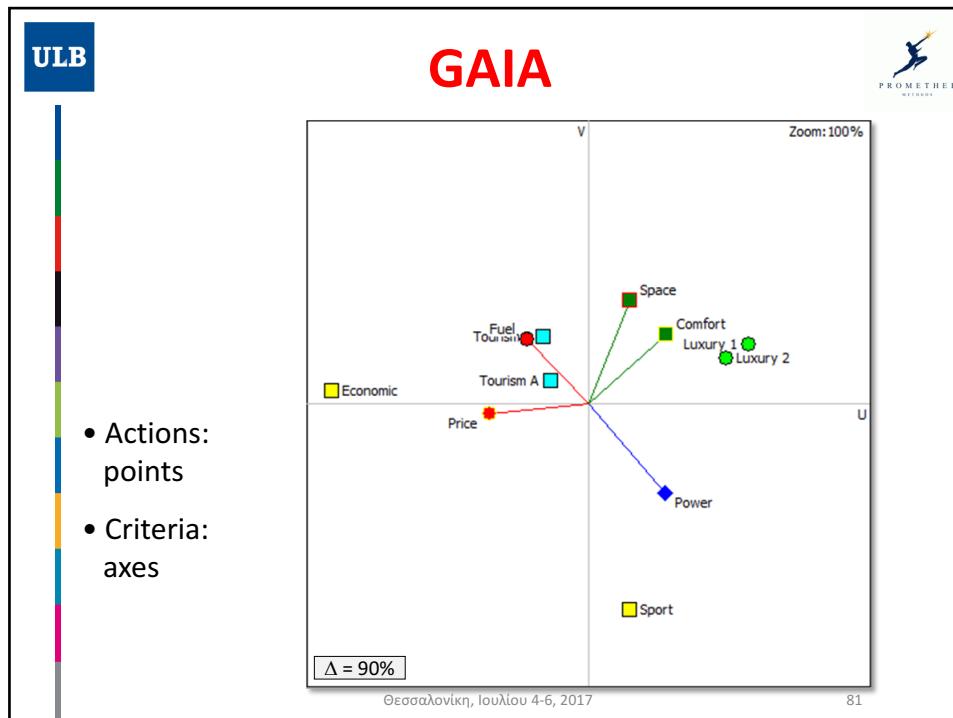
# GAIA

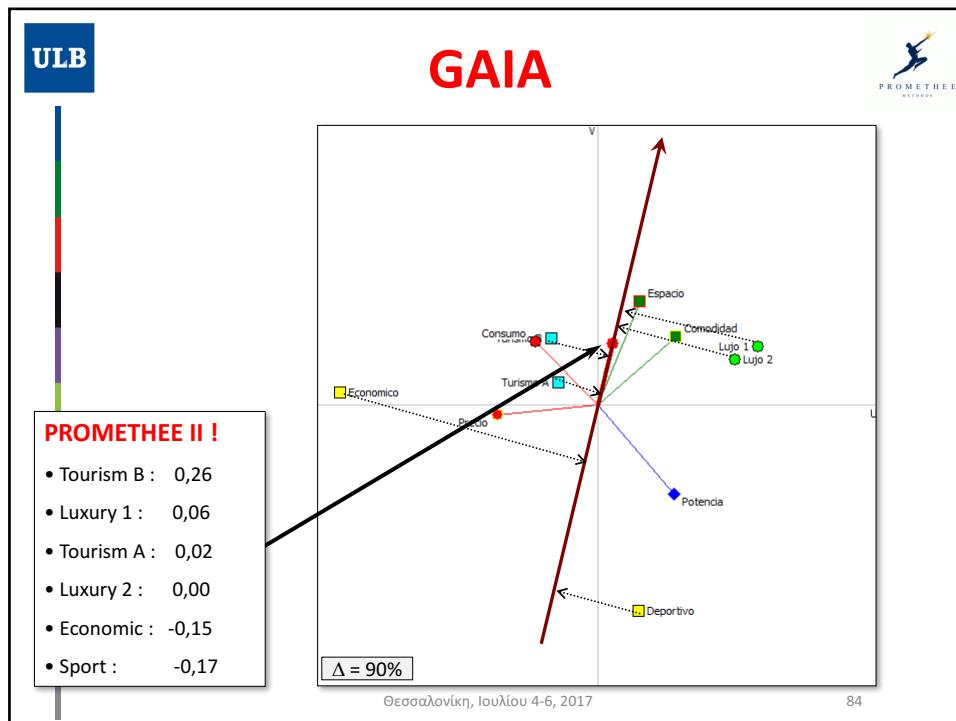
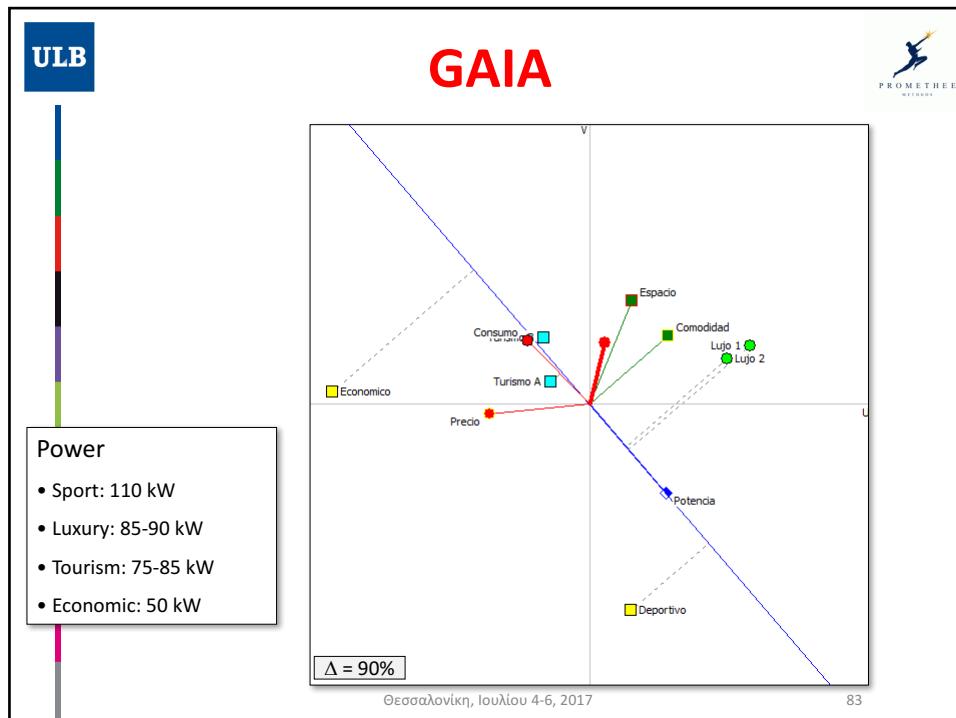
PROMETHEE METHODS

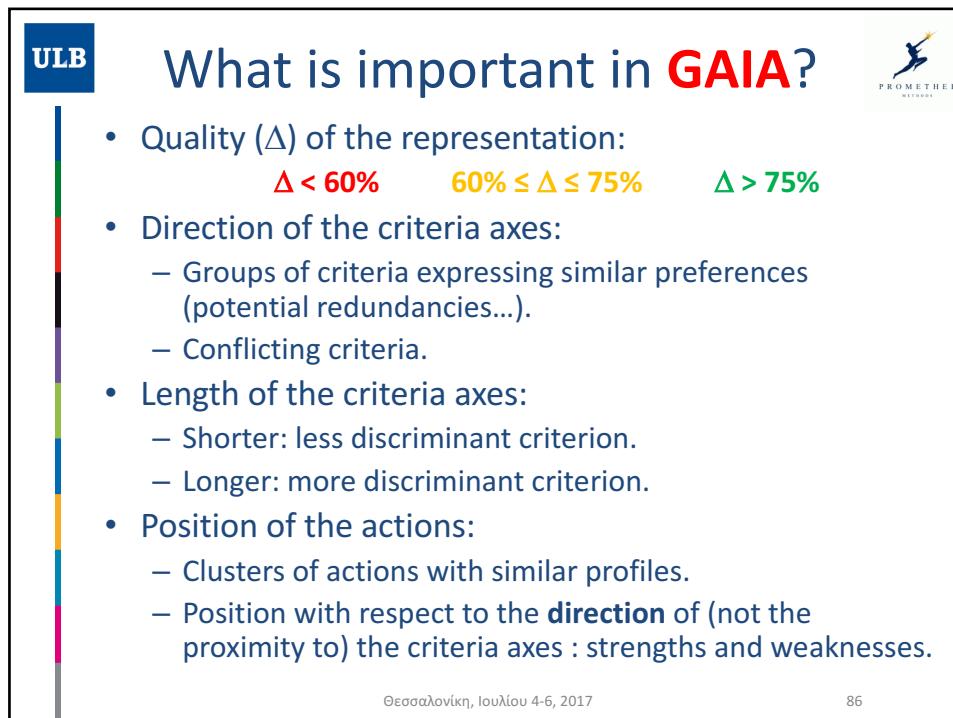
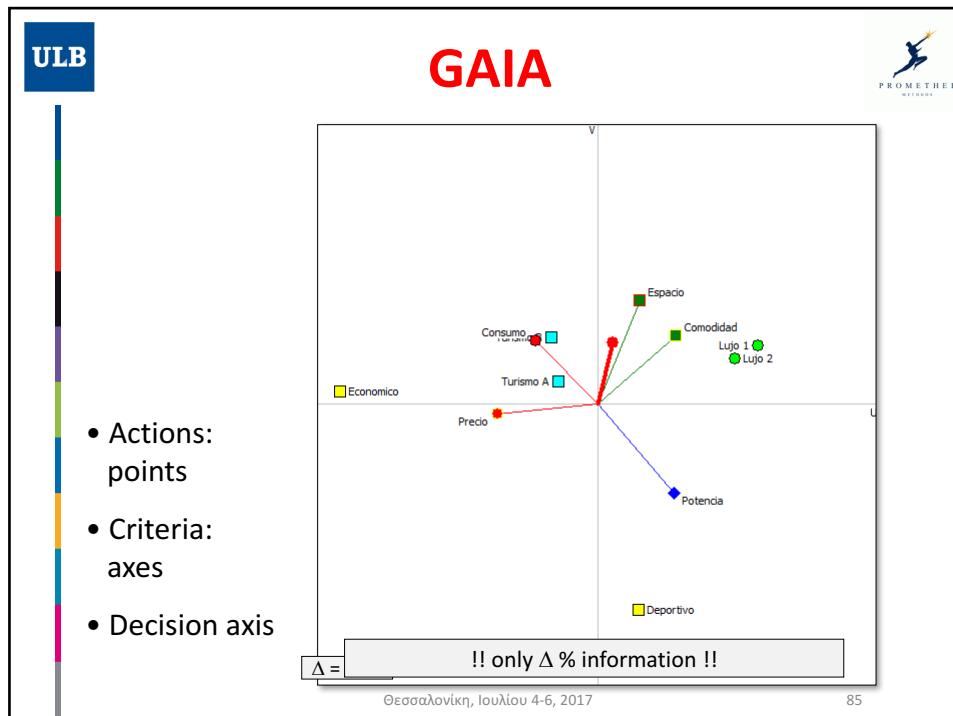
- Discover conflicts among criteria.
- Identify potential compromises.
- Help to fix priorities.

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## Sensitivity Analysis with **GAIA**



- Criteria weights ↔ Decision axis position.
- Interactive weight sensitivity analysis:  
« Walking Weights ».
- Robustness with respect to weight values?
  - Decision maker « brain » (**PROMETHEE VI**).
  - Area determined by the tip of the decision axis when criteria weights are changed within predefined percentages.

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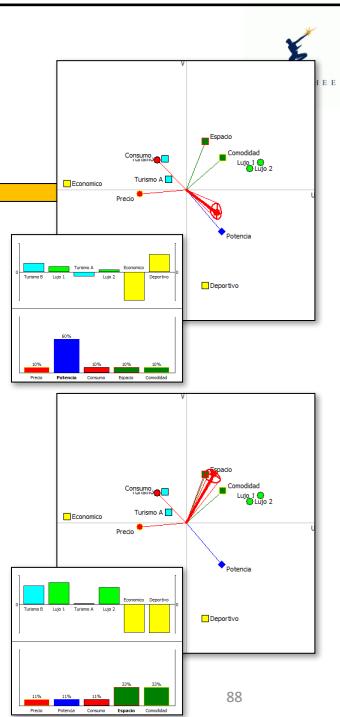
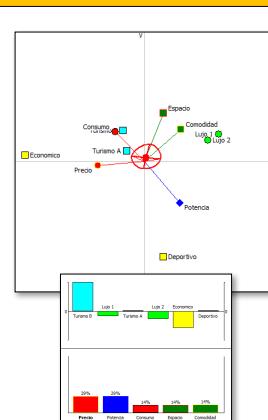
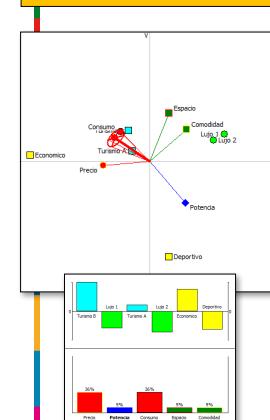
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## GAIA-Brain

20 years old

35 years old



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## Enhancing **GAIA**

PROMETHEE

- Limits of **GAIA**:
  - Imperfect view of the multicriteria data ( $\Delta\%$ ).
  - Potential distortion of the action profiles.
  - Non-optimal representation of the decision axis (weights) and distortion of the **PROMETHEE II** ranking (especially when the decision axis is shorter).
- New « **GAIA-type** » views:
  - **GAIA 3D**
  - **GAIA Webs**

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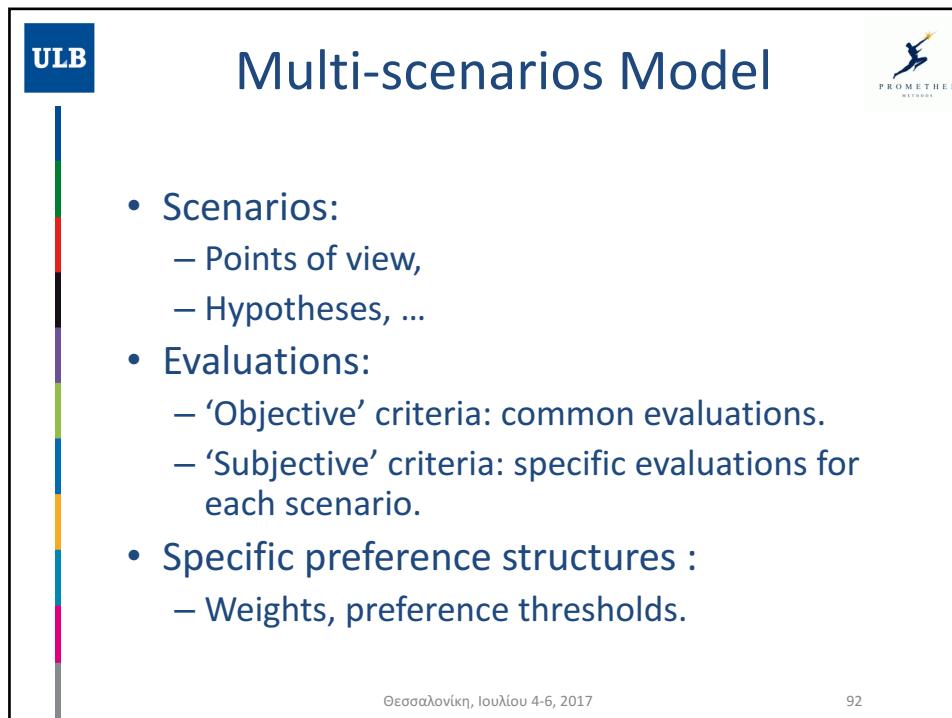
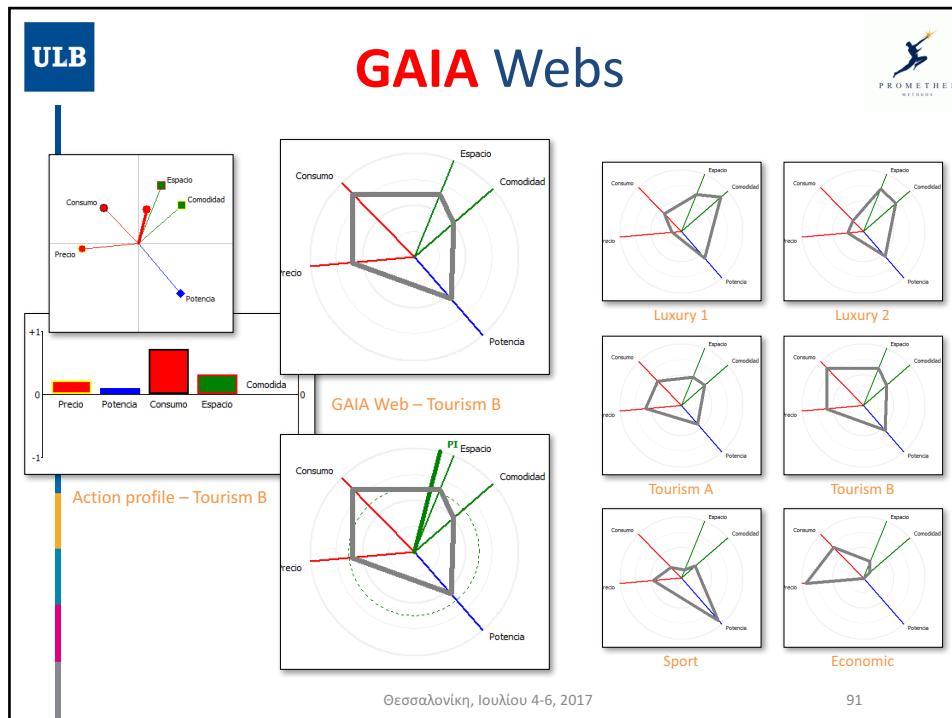
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## **GAIA 3D**

PROMETHEE

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## Multi-scenarios Model



- Adaptation of **PROMETHEE**:
  - Individual rankings,
  - Global (group) ranking with possible weighing of the scenarios.
- Adaptation of **GAIA**:
  - Three different analyses:
    - **GAIA**-Criteria,
    - **GAIA**-Scenarios,
    - **GAIA**-Unicriterion.

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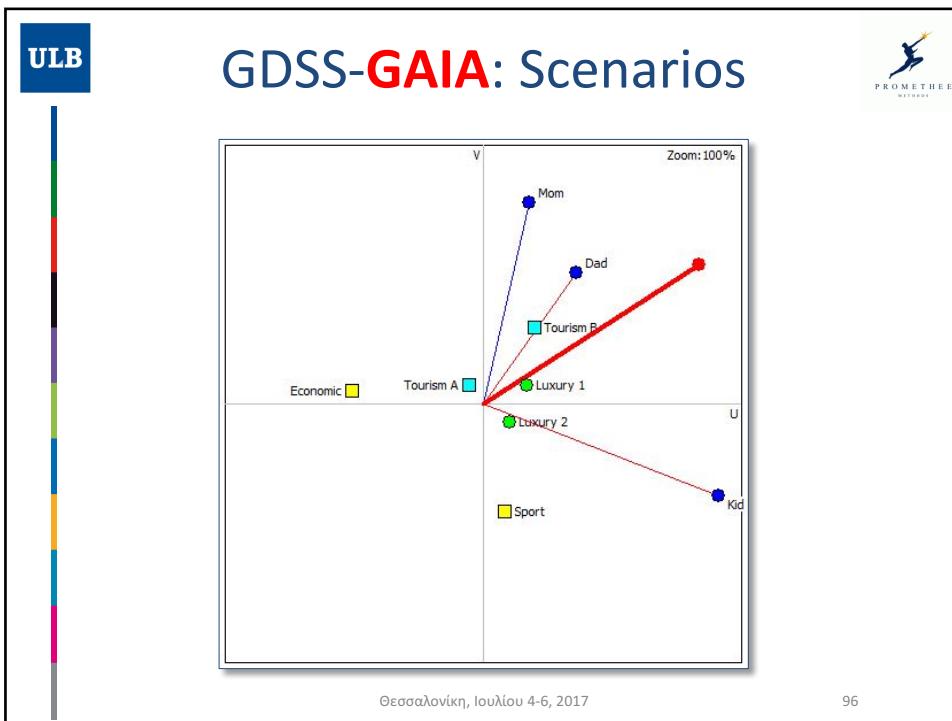
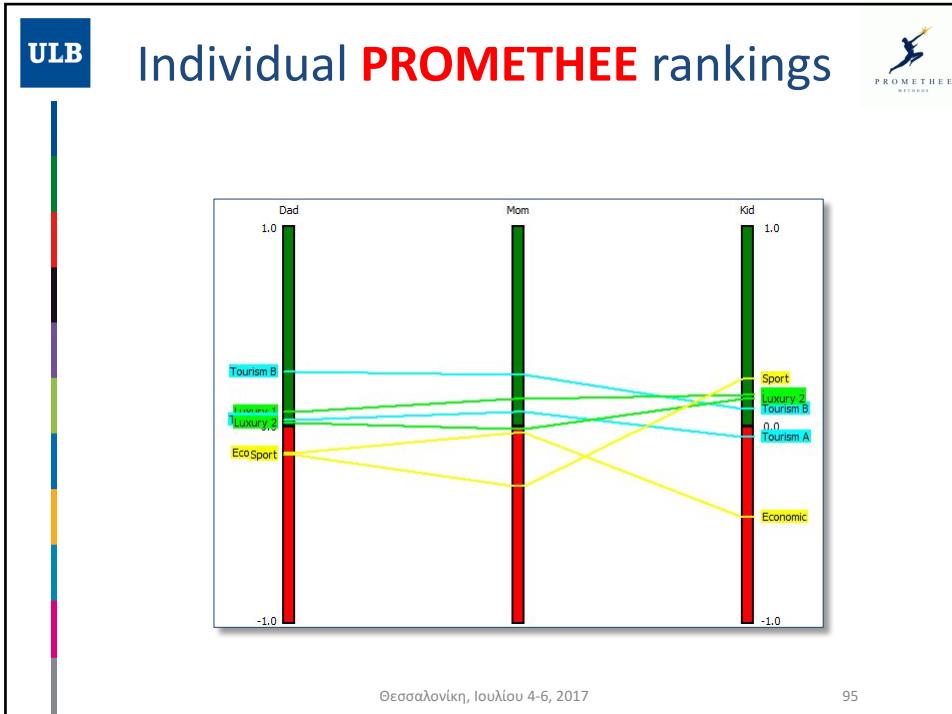
## Example

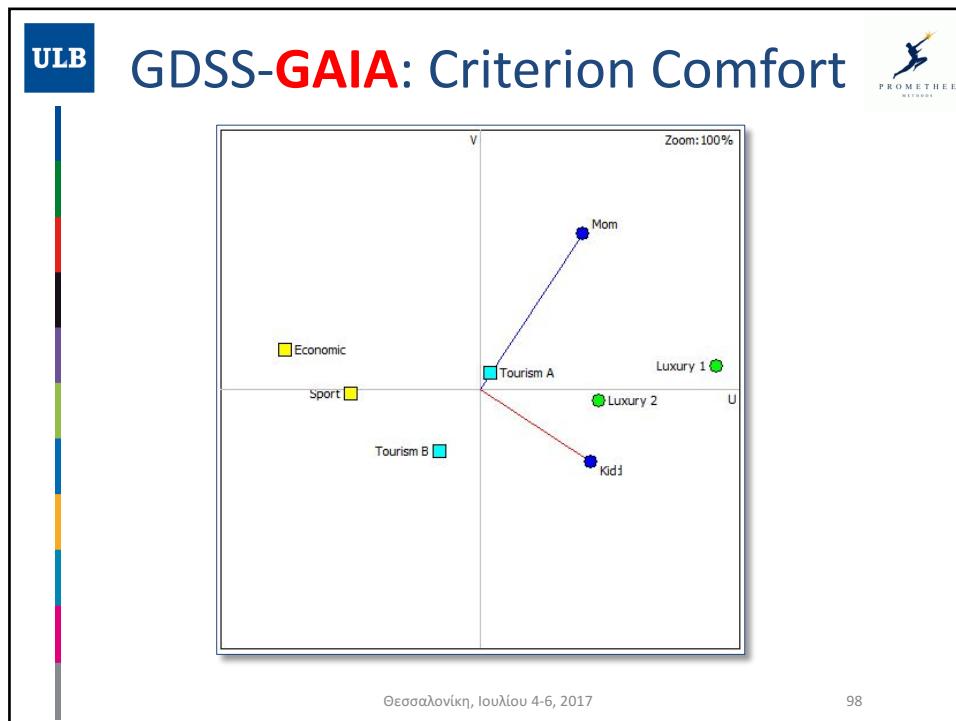
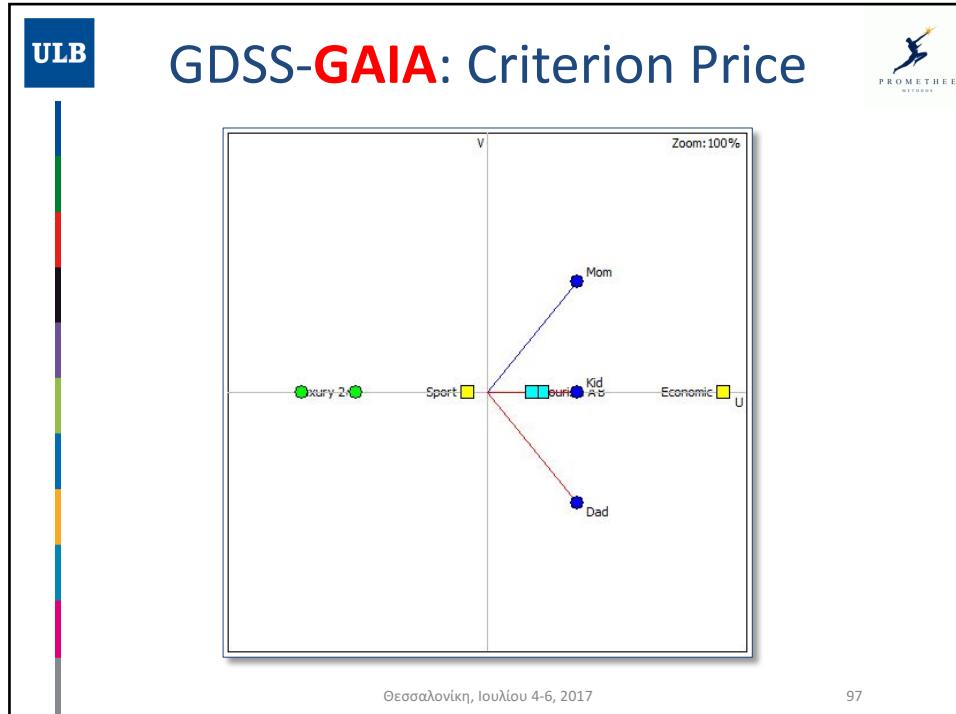


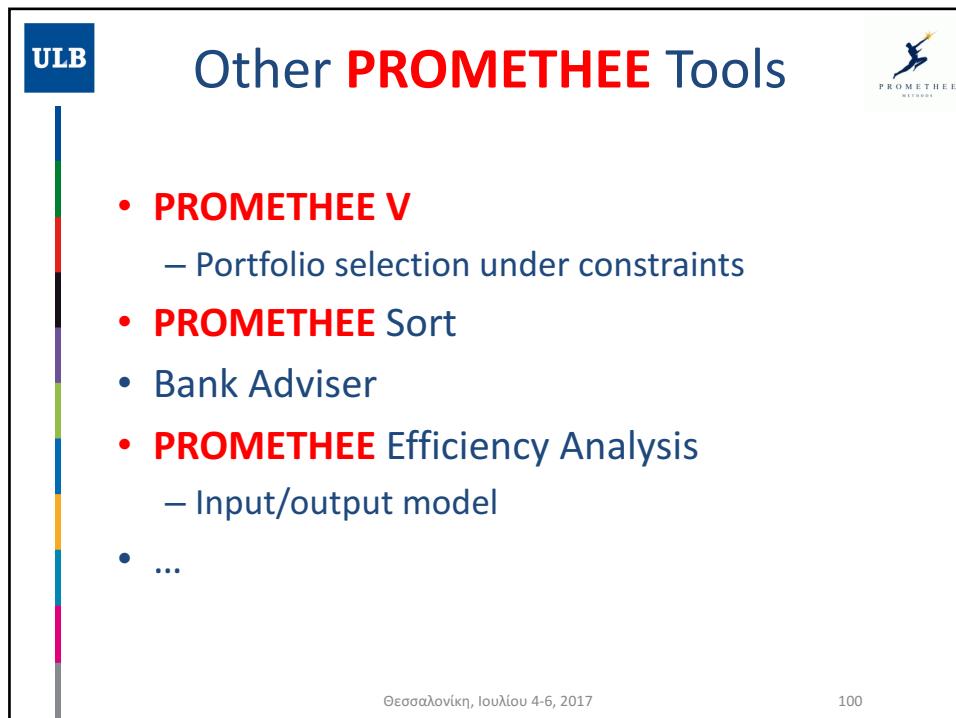
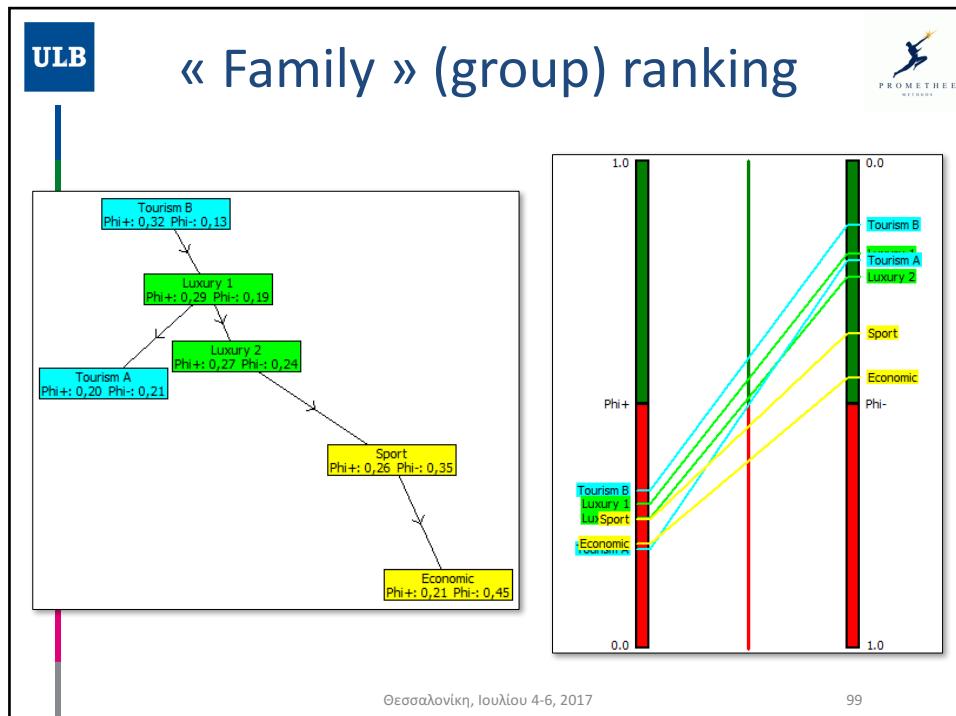
- A Greek family.
- Three actors (“decision makers”):
  - Ιάσων (dad),
  - Ελευθερία (mom),
  - Γιώργος (the kid).
- Three scenarios.
- Three multicriteria tables:
  - Different weights.
  - Subjective evaluation of comfort.

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## Google Maps interface

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PROMETHEE  
METHODS

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## Homework Assignment

PROMETHEE  
METHODS

- For Thursday.
- By small groups of 4 to 6 students.
- Set up a multicriteria decision problem:
  - Actual or fictive.
  - Including minimum 6 actions, 5 criteria and 2 scenarios.
  - Prepare the multicriteria evaluation table.
  - Think about preference functions and criteria weights.

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# Visual PROMETHEE

[WWW.PROMETHEE-GAIA.NET](http://WWW.PROMETHEE-GAIA.NET)



- **Visual PROMETHEE** software:
  - Free Academic Edition
  - Business Edition
- <http://biblio.promethee-gaia.net> :
  - Over 1570 references available.
- **Visual PROMETHEE** Manual (PDF or ebook)
- Services: Training, Coaching, Free seminars
- <http://blog.promethee-gaia.net>
- <http://www.promethee-days.com> Spring 2018
- LinkedIn group, Twitter, ResearchGate, ...

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## Thursday 6

### Practice



- Exercises.
- Case studies (homeworks).

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## Steps for using PROMETHEE

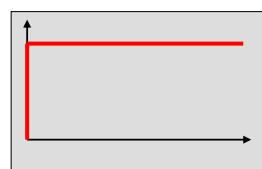


1. Define the actions (list)
2. Define the criteria
  - Quantitative
  - Qualitative (scale)
3. Fill the evaluation table (evaluations).
4. For each criterion:
  - Choose the right type of preference function
  - Set the appropriate thresholds
5. Set the weights of the criteria

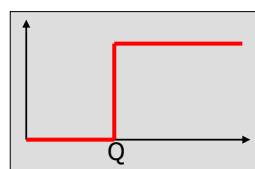
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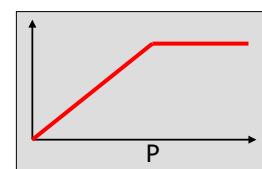
## Preference Functions (as in Visual PROMETHEE software)



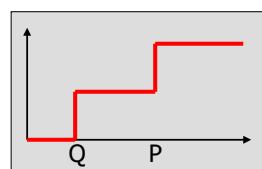
Usual



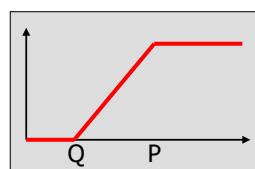
« U » shape



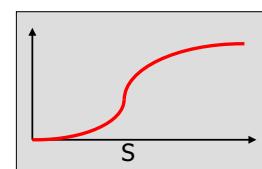
« V » shape



Level



Linear



Gaussian

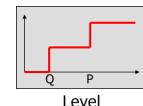
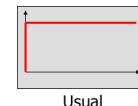
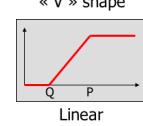
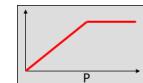
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## Preference Functions



- For continuous quantitative criteria (e.g. cost, price, power):
  - V-shape (no indifference threshold)
  - Linear
- For qualitative or discrete quantitative criteria (e.g. « very good to very bad », number of USB ports):
  - Usual (no thresholds)
  - Level



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## Think about...



- Using action categories:
  - To identify similar actions by color.
- Using clusters and groups of criteria:
  - Two-level hierarchy:
    - Clusters are main families of criteria (economy, environment, social, ...),
    - Groups are subsets of related criteria within a cluster (air, water, landscape, ... within environment).
  - Color code.
  - Grouping for easier sensitivity analysis.

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## Next...



- **First:** Thanks to all of You!
- **Continue...**
- Check <http://www.promethee-gaia.net>
- Join **PROMETHEE** methods LinkedIn group
- Download **Visual PROMETHEE**
  - Free full-featured Academic Edition
- Interact with us:
  - **PROMETHEE Academy**
  - **PROMETHEE Days 2018** in Heraklion!  
<http://www.prometheedays.com>

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