Summary

1. Introduction
   - Statistics, management science, operations research, decision aid, …
2. Decision analysis
3. Advanced optimization
   - Linear programming
   - Integer programming
   - Non-linear programming
4. Networks
   - Transportation problems
   - Network flow problems
   - Project management
5. Inventory management
6. Simulation models
7. Advanced statistical methods
   - Data mining
   - Analysis of variance
   - Forecasting

Network

- Arrangement of paths (branches) connected at various points (nodes) through which items move.
- Nodes: junction points connecting branches.
- Branches: lines, connect nodes, show flow from one node to another.

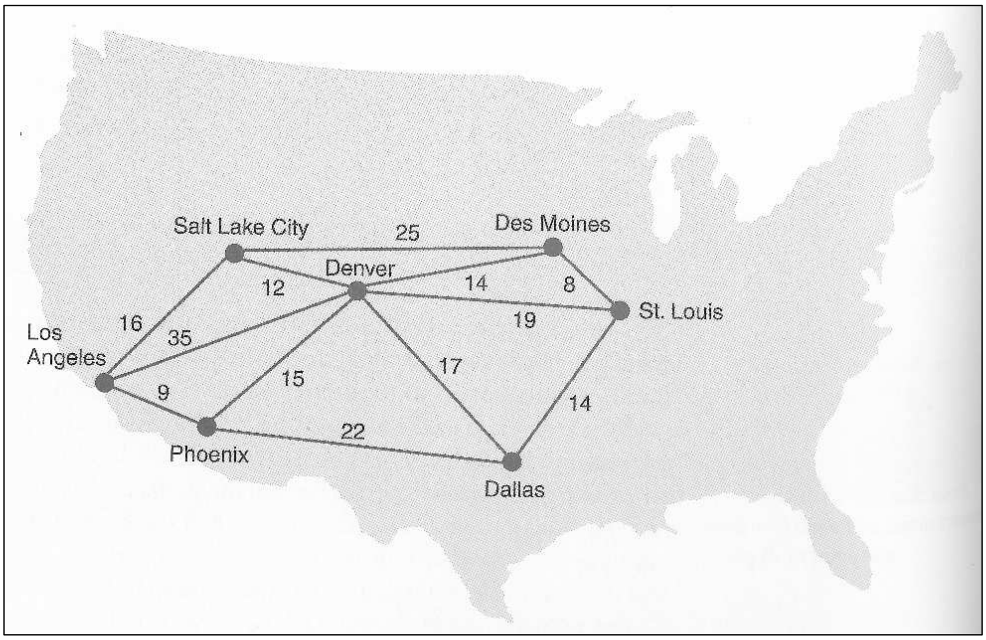
Network flow models

1. Shortest route problem.
2. Minimal spanning tree problem.
3. Maximal flow problem.

Shortest route problem

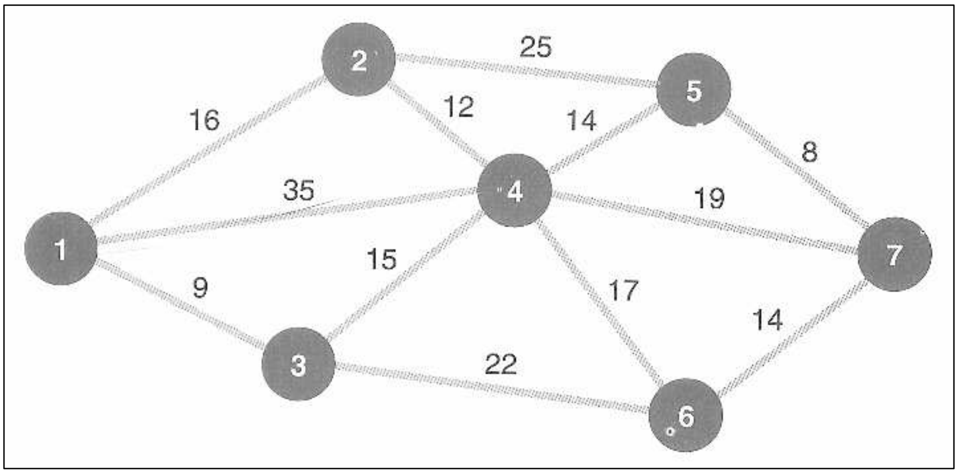
- What is the shortest distance between an originating point and several destination points ?
- Example: Stagecoach Shipping Company
  - 6 trucks transport oranges from LA to 6 cities.
  - What are the best (shortest travel time) routes ?

Stagecoach Shipping Company



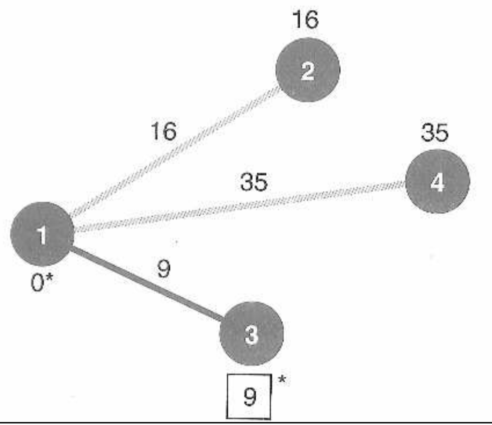
Network representation

- Determine the shortest routes from node 1 to nodes 2 through 7.



Solution (1)

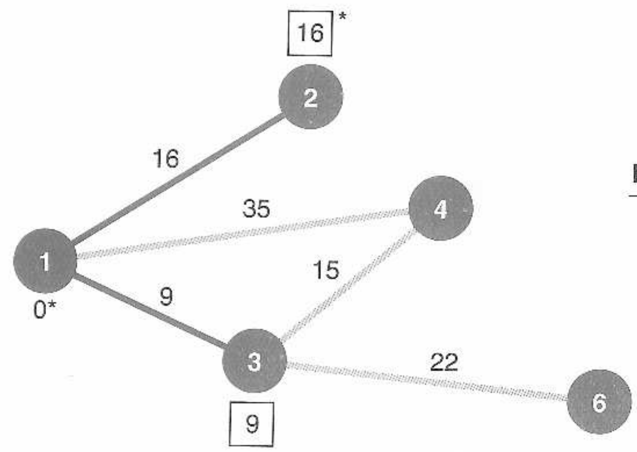
- Start with node 1 (first node in permanent set).
- Determine shortest time to get to a directly connected (adjacent) node. Node 3 gets in the permanent set.



Permanent set	Branch	Time
{1}	1–2	16
	1–4	35
	1–3	9 *

Solution (2)

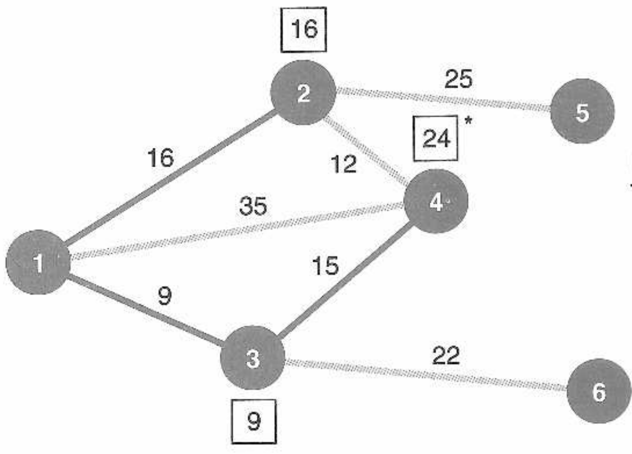
- Determine all nodes directly connected to nodes in the permanent set (2, 4 and 6).
- Determine the shortest route to these nodes. Node 2 becomes part of the permanent set.



Permanent set	Branch	Time
{1, 3}	1–2	16 *
	1–4	35
	3–4	24
	3–6	31

Solution (3)

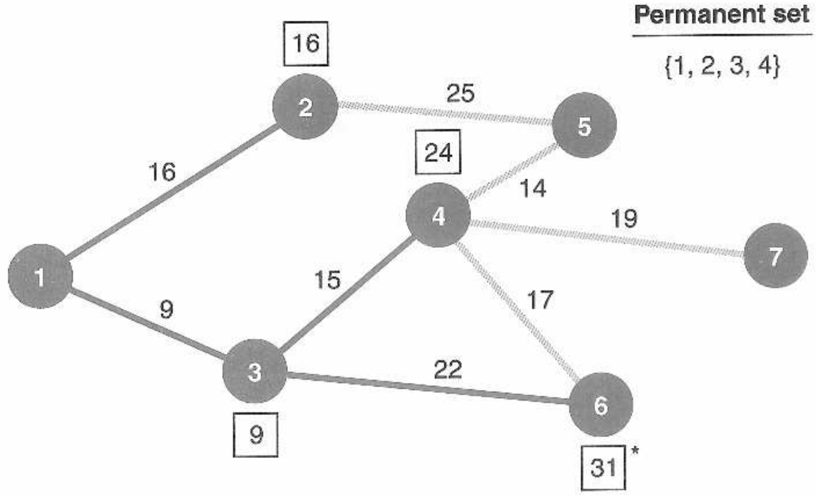
- Next step…



Permanent set	Branch	Time
{1, 2, 3}	1–4	35
	2–4	28
	2–5	41
	3–4	24 *
	3–6	31

Solution (4)

- Next step…

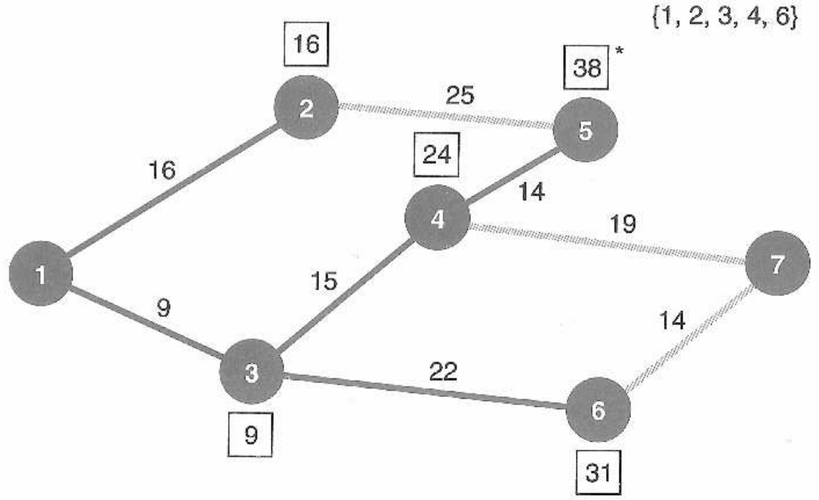


Permanent set	Branch	Time
{1, 2, 3, 4}	2–5	41
	3–6	31 *
	4–5	38
	4–7	43
	4–6	41

Solution (5)

- Next step…

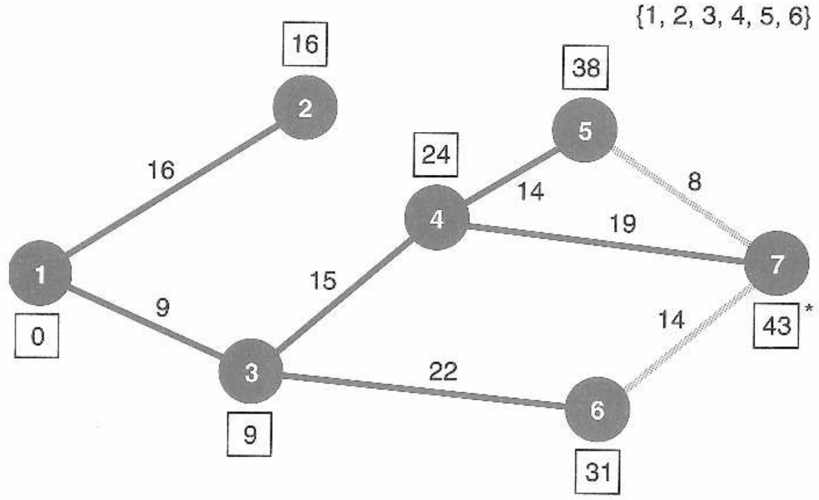
Permanent set	Branch	Time
{1, 2, 3, 4, 6}	2–5	41
	4–5	38 *
	4–7	43
	6–7	45



Solution (6)

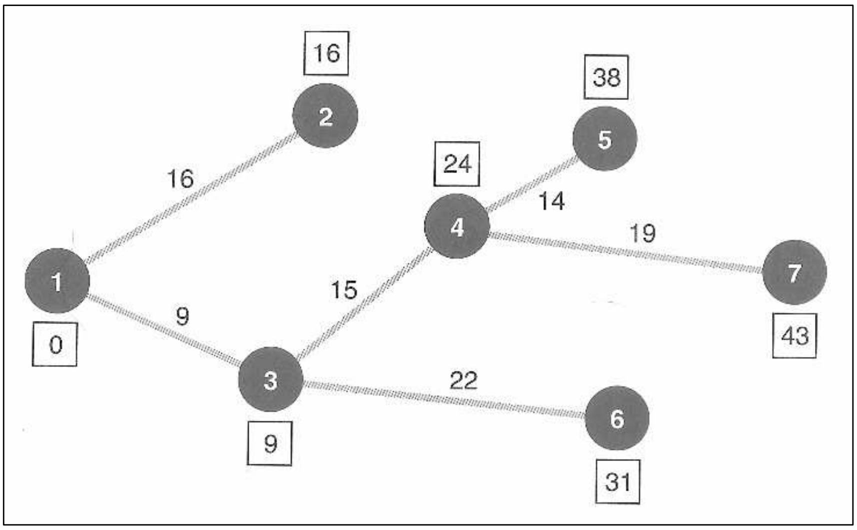
- Next step…

Permanent set	Branch	Time
{1, 2, 3, 4, 5, 6}	4–7	43 *
	6–7	45
	5–7	46



Solution (final)

- Optimal routes from LA to other cities:



Summary

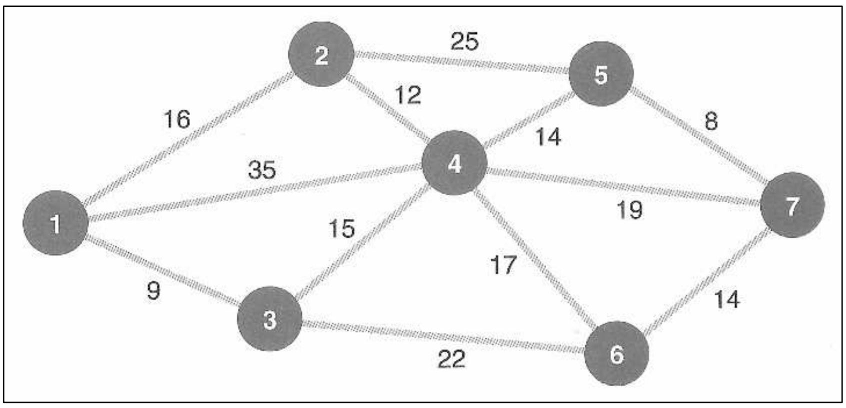
1. Select the node with the shortest direct route from the origin.
2. Establish a permanent set with the origin node and the node selected in step 1.
3. Determine all nodes directly connected to the permanent set nodes.
4. Select the node with the shortest route from the group of nodes directly connected to the permanent set nodes. This node joins the permanent set.
5. Repeat steps 3 and 4 until all nodes have joined the permanent set.

Minimal spanning tree

- To connect all nodes in a network so that the total branches lengths are minimized.
- Example: Metro Cable TV Company
  - To install a TV cable system in a community consisting of 7 suburbs.
  - Each suburb must be connected to the main cable system, with a minimal total length of cable.

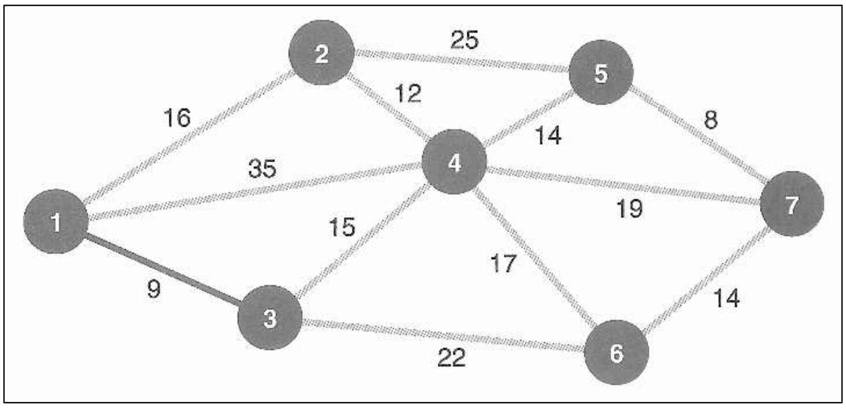
Network representation

- Nodes = suburbs.
- Branches = possible links (value = cable length in .000ft).



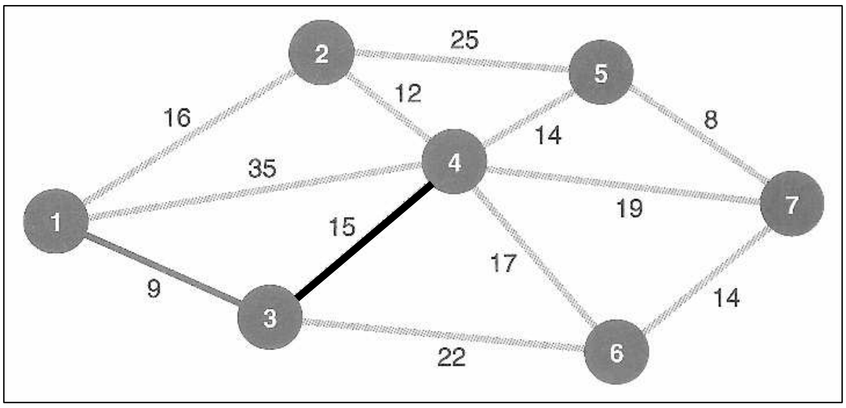
Solution (1)

- Start from node 1 (or any other).
- Select the closest node to node 1 (node 3).
- Branch from node 1 to node 3 belongs to the current spanning tree.



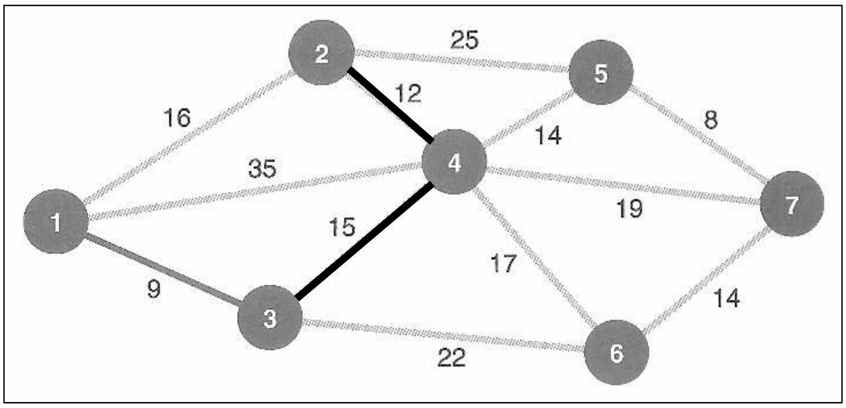
Solution (2)

- Select closest node to current spanning tree (node 4).
- Current spanning tree include nodes 1, 3 and 4.



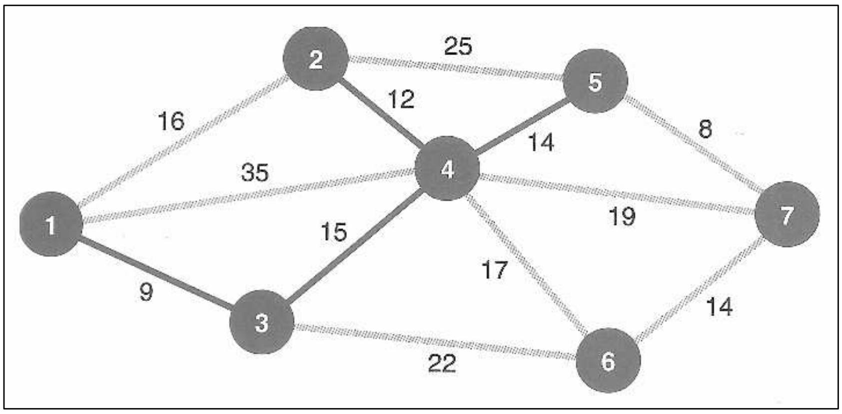
Solution (3)

- Continue…



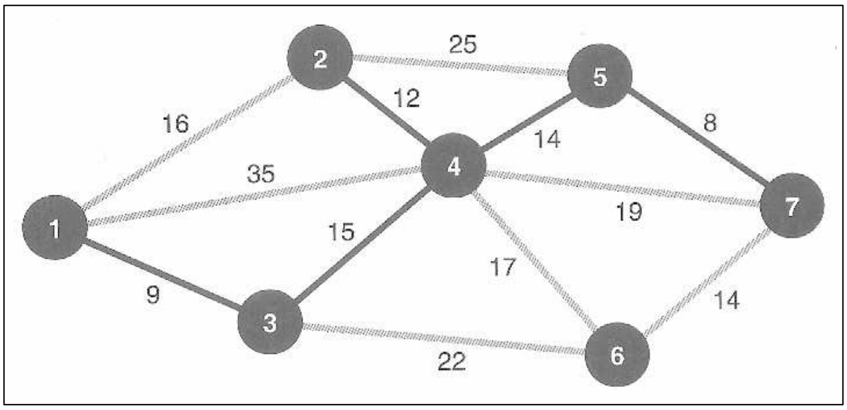
Solution (4)

- Continue…



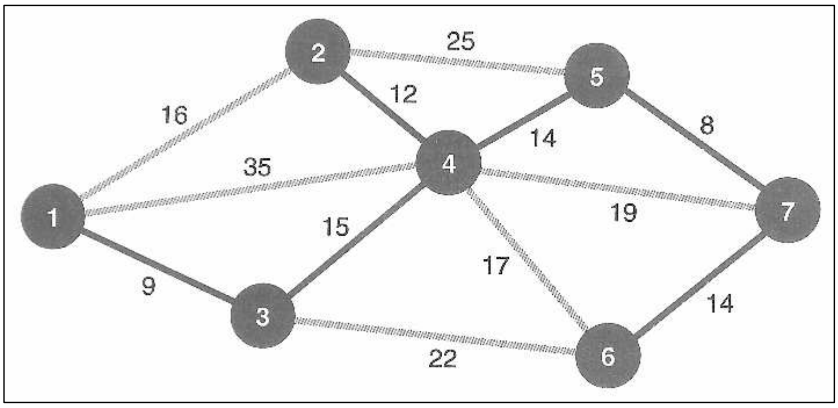
Solution (5)

- Continue…



Solution (6)

- Continue…



Summary

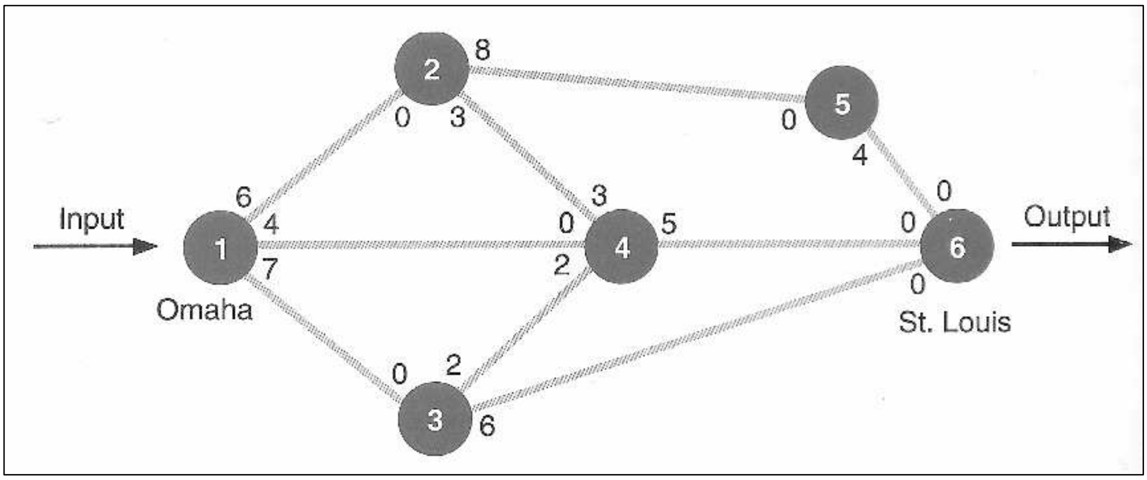
1. Select any starting node.
2. Select the node closest to the starting node to join the spanning tree.
3. Select the closest node not presently in the spanning tree.
4. Repeat step 3 until all nodes have joined the spanning tree.

Maximal flow problem

- Branches have limited flow capacities.
- Maximize the total amount of flow from an origin to a destination.
- Flow: water, gas, oil, traffic, information, …
- Example: Scott Tractor Company
  - Railway system between Omaha and St. Louis.
  - Shipping tractor parts from Omaha to St. Louis.
  - Limited number of railroad cars available on each branch of the network during a week.

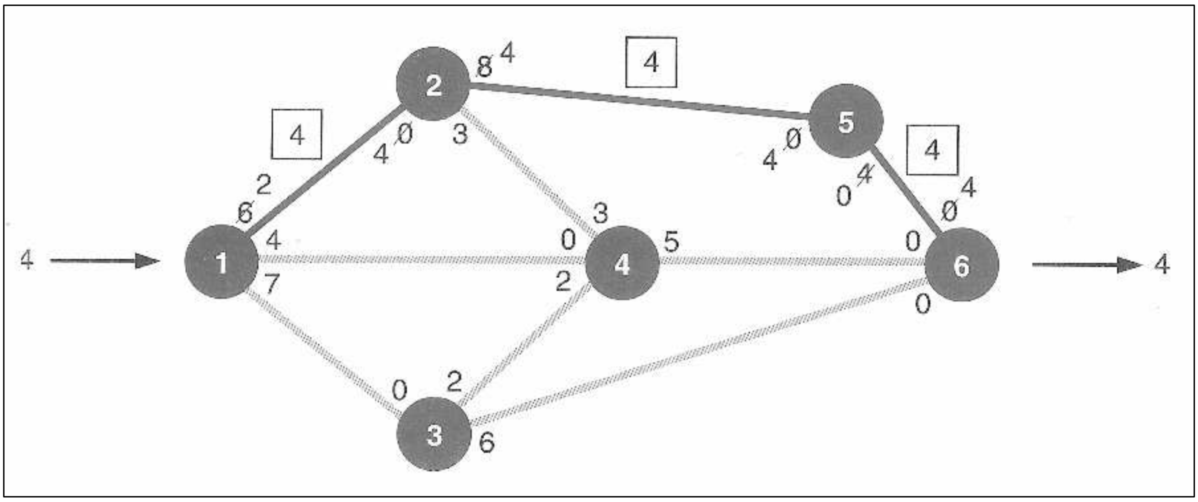
Network representation

- Directed branches!



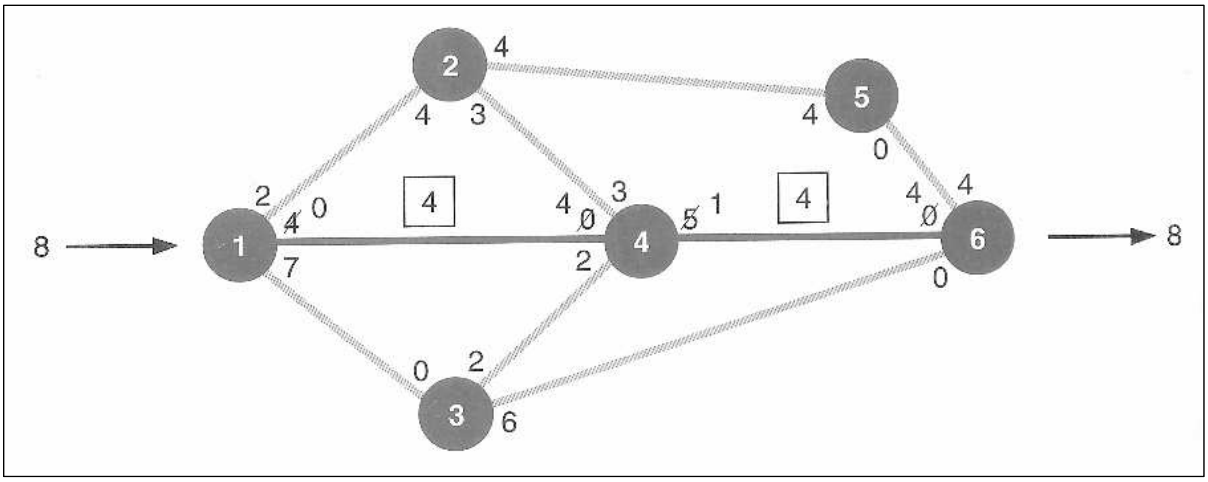
Solution (1)

- Choose any path (1-2-5-6) from origin to destination and ship as much as possible (4 cars) through that path.
- Adjust remaining capacities.



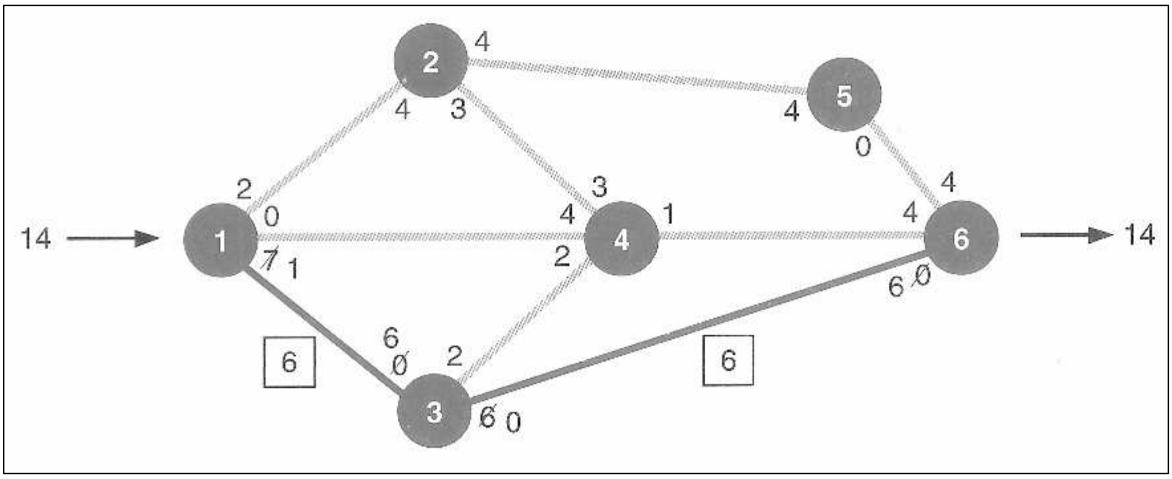
Solution (2)

- Select another path… 1-4-6 $\rightarrow$ 4 cars.



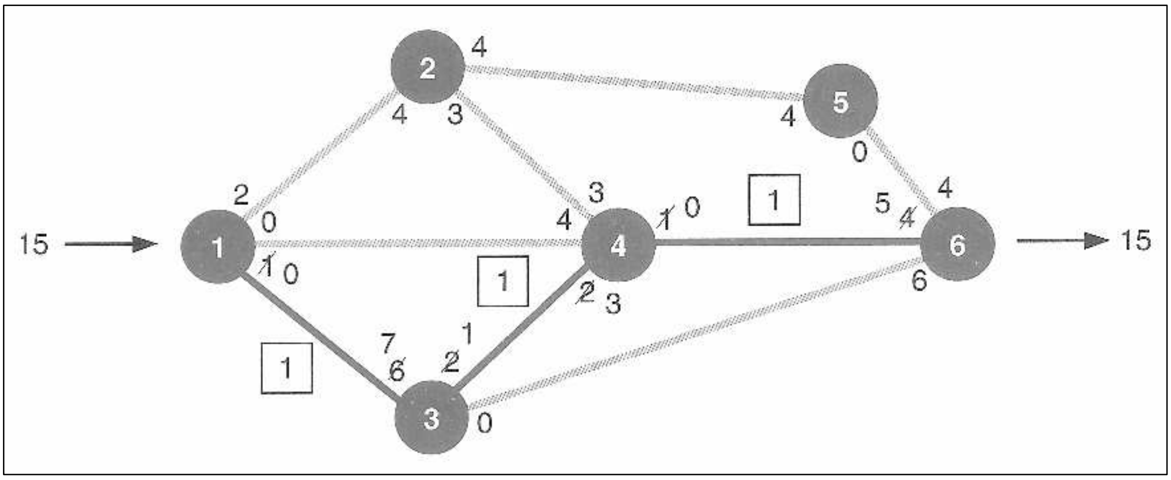
Solution (3)

- Select another path… 1-3-6 $\rightarrow$ 6 cars.



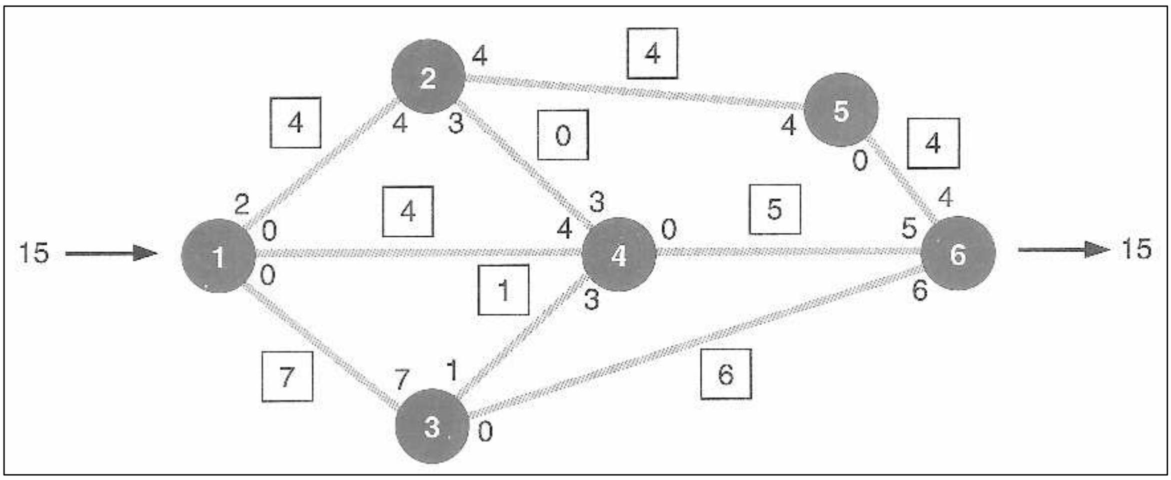
Solution (4)

- Select another path… 1-3-4-6 → 1 car.



Solution (final)

- No more path available !
- Maximal flow of 15 cars !



Summary

1. Select any path in the network from origin to destination.
2. Adjust the capacities at each node by subtracting the maximal flow for the path selected in step 1.
3. Add the maximal flow along the path to the flow in the opposite direction at each node.
4. Repeat steps 1, 2 and 3 until there are no more paths with available flow capacity.

Project management

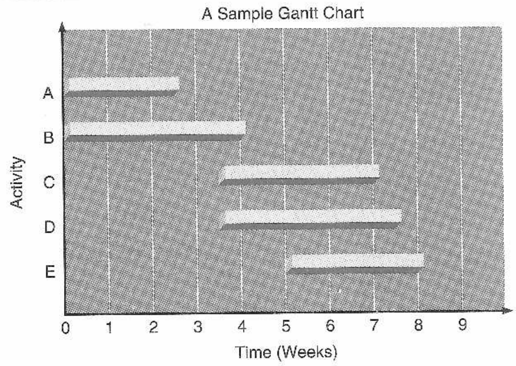
- Projects ?
  - Building, product development, ERP implementation, going to Mars, …
- PERT (Program Evaluation and Review Technique)
- CPM (Critical Path Method)
- Purpose:
  - Help managers to plan, schedule, monitor and control large and complex projects.

Starting point

HISTORY **How PERT and CPM Started**

Managers have been planning, scheduling, monitoring, and controlling large-scale projects for hundreds of years, but it has only been in the past 50 years that QA techniques have been applied to major projects. One of the earliest techniques was the *Gantt chart*. This type of chart shows the start and finish times of one or more activities, as shown in the accompanying chart.

In 1958, the Special Projects Office of the U.S. Navy developed the Program Evaluation and Review Technique (PERT) to plan and control the Polaris missile program. This project involved the coordination of thousands of contractors. Today PERT is still used to monitor countless government contract schedules. At about the same time (1957), the Critical Path Method (CPM) was developed by J. E. Kelly of Remington Rand and M. R. Walker of du Pont. Originally, CPM was used to assist in the building and maintenance of chemical plants at du Pont.



Six steps

1. Define project and all activities (tasks).
2. Decide which tasks must precede and follow others.
3. Draw the network.
4. Assign time and/or cost estimates to each task.
5. Find the **critical path** through the network.
6. Use the network to help plan, schedule, monitor and control the project.

PERT vs CPM

- PERT:
  - Probabilistic approach: task durations are not known with certainty – three estimates are associated to each task (optimistic, pessimistic, mos likely).
- CPM:
  - Deterministic approach: two durations are associated to each task (normal time and crash time).

PERT – questions ?

- When will the project be completed ?
- What are the critical activities (delays) ?
- What is the probability to finish at a given date ?
- Are we on/behind/ahead of schedule ?
- Are there enough resources available to finish on time ?
- …

General Foundry Example

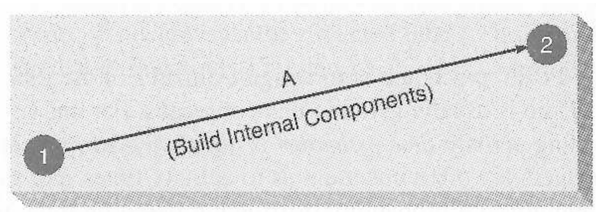
- General Foundry Inc. (metalworks) has 16 weeks to install a complex air pollution control equipment in its plant.
- The project can be split into 8 separate tasks to perform.

Tasks and constraints

ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	A
D	Pour concrete and install frame	B
E	Build high-temperature burner	C
F	Install control system	C
G	Install air pollution device	D, E
H	Inspect and test	F, G

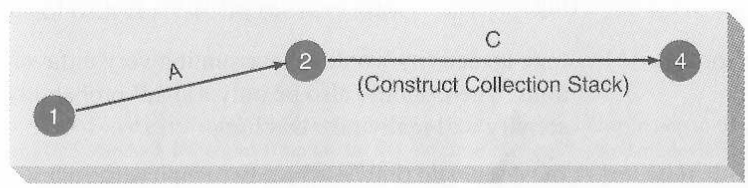
PERT network

- Each task:
  - Is represented by an arrow (branch).
  - Uses time and resources (work, budget, …).
- The nodes of the network correspond to **events**:
  - Start or completion of one or several tasks.



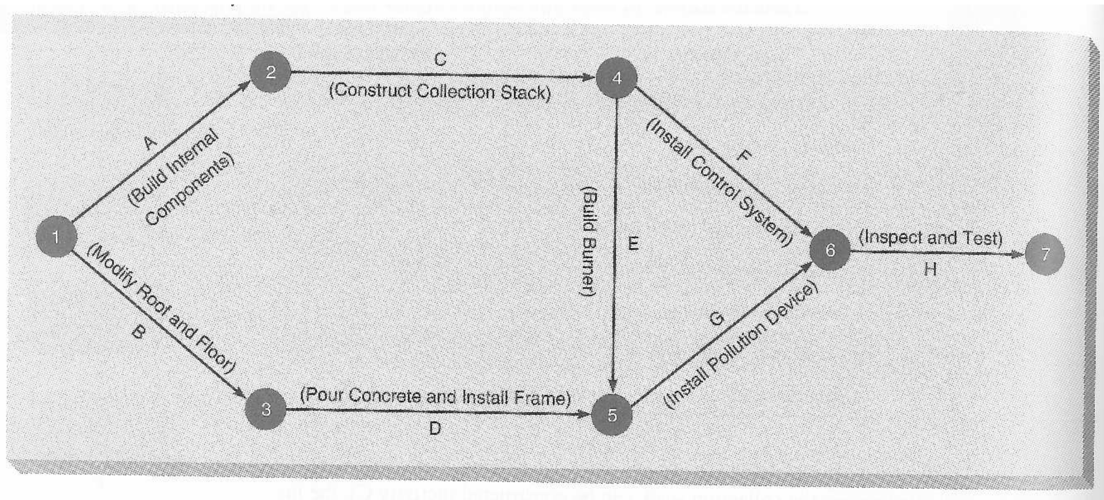
PERT network

- Task A is the only predecessor of C :



- Drawing the network:
  - Starting from the tasks that have no predecessors.
  - From the start of the project (node 1) to the end of the project.

PERT network for General Foundry

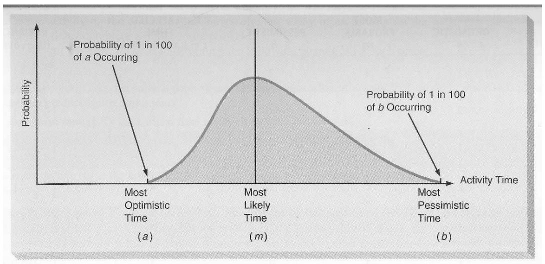


Activity times

- Not known with certainty.
- Probability distribution based on three time estimates (for each task):
  - Optimistic time (a): time if everything goes as well as possible, not so likely (prob ≈ 1%).
  - Pessimistic time (b): time assuming most unfavorable conditions, not so likely (prob ≈ 1%).
  - Most likely time (m): most realistic time estimate.

Beta probability distribution

- Assumption appropriate in many cases.



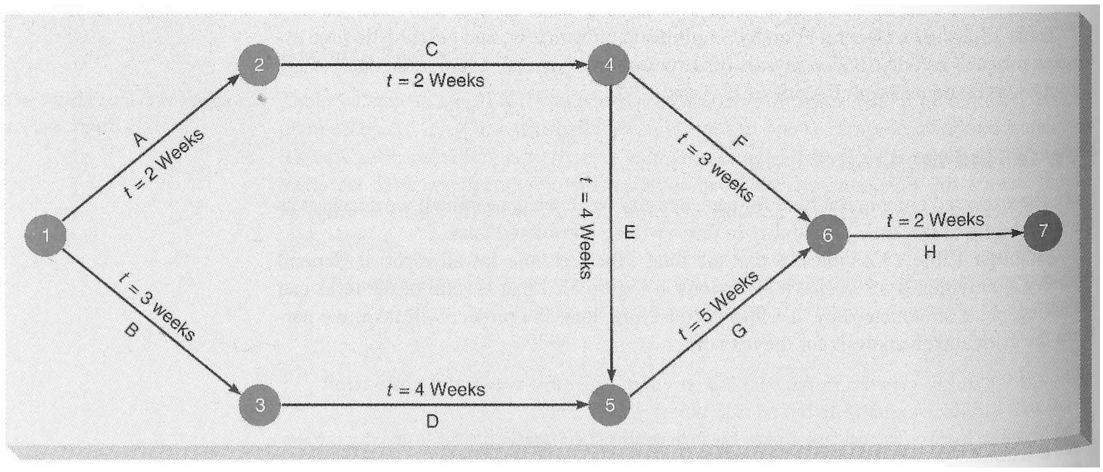
- Expected time (mean): $t = \frac{a + 4m + b}{6}$
- Variance: $\text{var} = \left(\frac{b - a}{6}\right)^2$

General Foundry time estimates (in weeks)

ACTIVITY	OPTIMISTIC, a	MOST PROBABLE, m	PESSIMISTIC, b	EXPECTED TIME, $t = [(a + 4m + b)/6]$	VARIANCE, $[(b - a)/6]^2$
A	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$
B	2	3	4	3	$\left(\frac{4-2}{6}\right)^2 = \frac{4}{36}$
C	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$
D	2	4	6	4	$\left(\frac{6-2}{6}\right)^2 = \frac{16}{36}$
E	1	4	7	4	$\left(\frac{7-1}{6}\right)^2 = \frac{36}{36}$
F	1	2	9	3	$\left(\frac{9-1}{6}\right)^2 = \frac{64}{36}$
G	3	4	11	5	$\left(\frac{11-3}{6}\right)^2 = \frac{64}{36}$
H	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$

Finding the critical path

- Assume the actual time of a task is its expected completion time (t).



Critical path

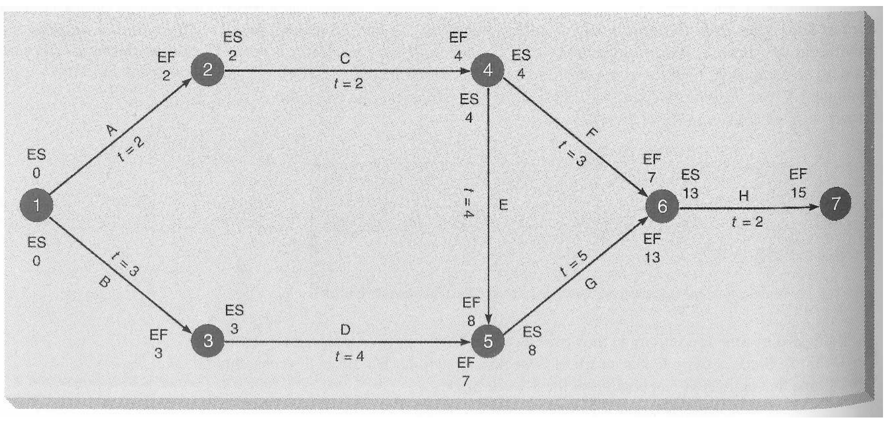
- Longest time path in the network from the start to the end of the project.
- Length = minimum completion time for the project.
- To reduce the total project time: reduce the length of the critical path.
- Any delay of a critical task will delay the entire project.

Computation

- For each task (assuming project start at time 0):
  - Earliest start time (ES): earliest time a task can begin without violation of immediate predecessors requirements.
  - Earliest finish time (EF): ES + t
  - Latest finish time (LF): latest time a task can end without delaying the project.
  - Latest start time (LS): LF - t

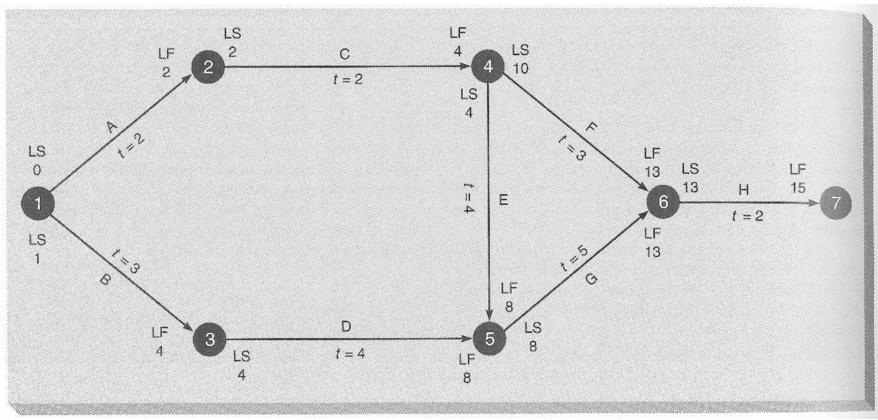
Computation ES - EF

- Starting from node 1.
- For any task, ES is equal to the length of the longest path from node 1 to the start of the task.
- Forward pass in the network.

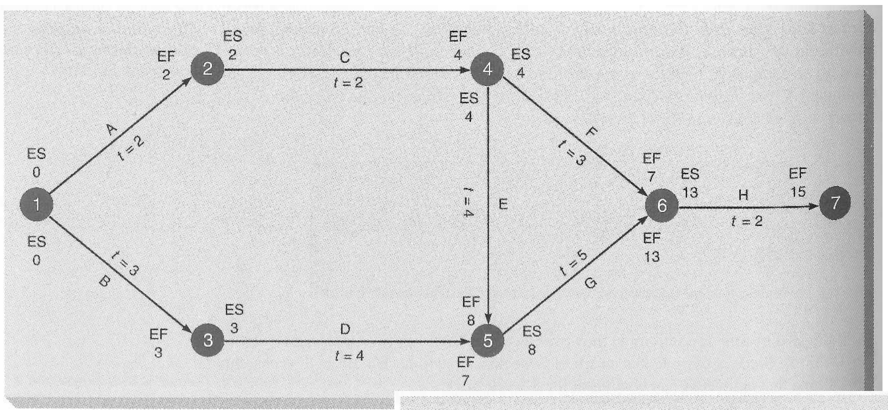


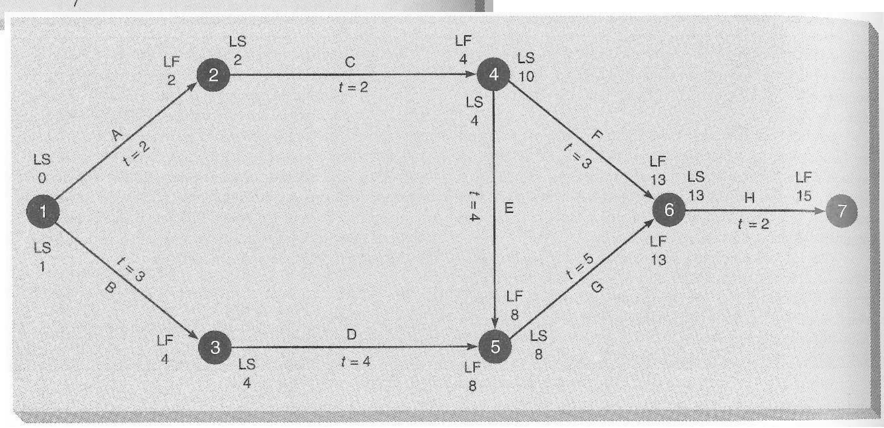
Computation LF - LS

- Starting from the last node (7).
- For any task, LF is computed first and is the latest time the task can finish without delaying the project.
- Backward pass in the network.



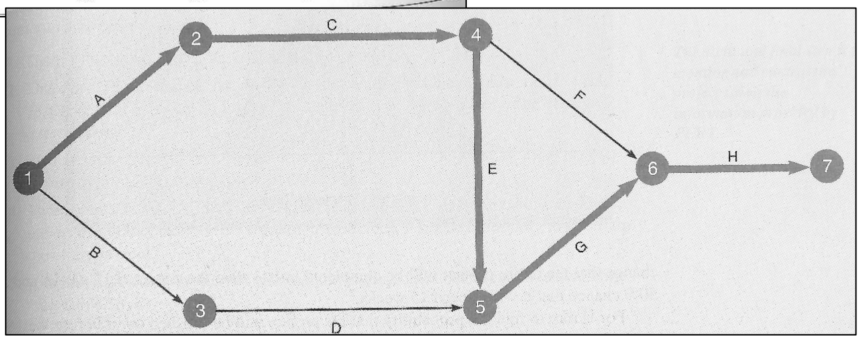
Slacks and critical path





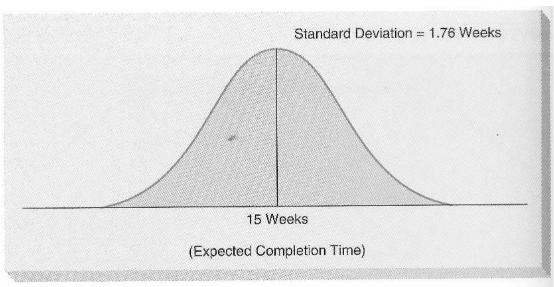
Slacks and critical path

ACTIVITY	EARLIEST START, ES	EARLIEST FINISH, EF	LATEST START, LS	LATEST FINISH, LF	SLACK, LS – ES	ON CRITICAL PATH?
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes



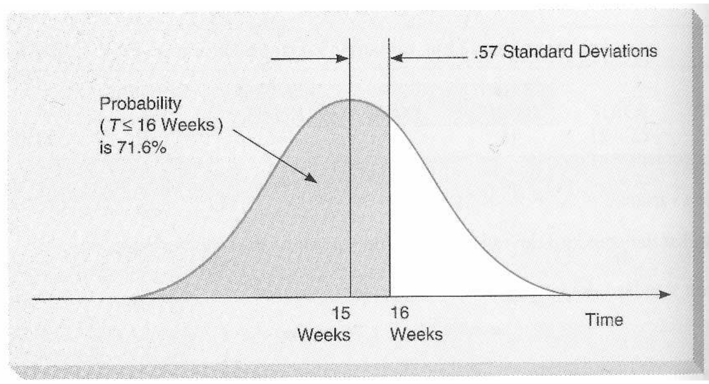
Probability of project completion

- Expected completion time is 15 weeks.
- Length of critical path = sum of durations of critical tasks = random variable !
- Two additional assumptions:
  - Total project time is normally distributed.
  - Activity times are independent random variables.



Probability of project completion

- Critical tasks: A, C, E, G, H
- Variance of project completion time: sum of the variances of the critical tasks.

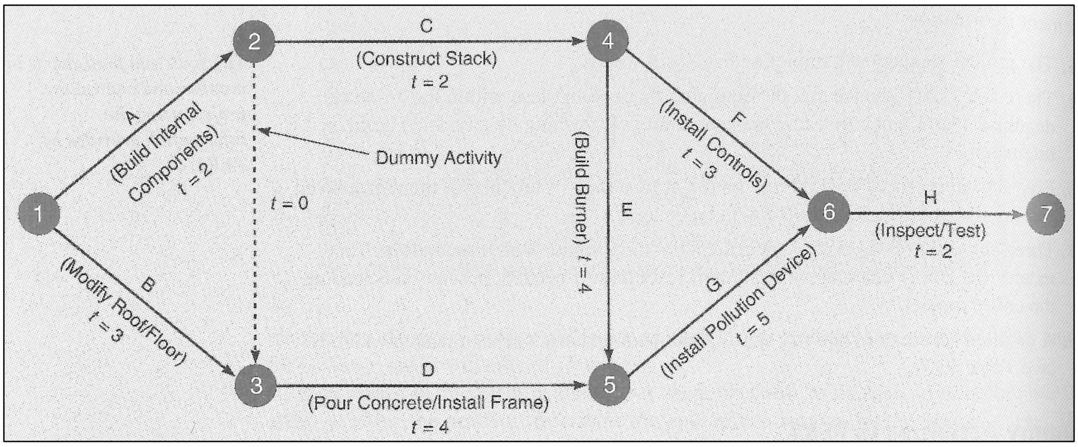


Conclusion

- Expected completion date is 15 weeks.
- 71,6% chance that the project is completed within 16 weeks.
- Five critical tasks (A, C, E, G, H) to monitor closely.
- Three non critical tasks (B, D, F) with corresponding slacks.
- Detailed schedule.

Dummy activities in PERT

- Let us assume that A has also to be completed before D can begin.



PERT/Cost

- What about project cost ?
- Each task has a cost.

ACTIVITY	EARLIEST START TIME, ES	LATEST START TIME, LS	EXPECTED TIME, t	TOTAL BUDGETED COST ($)	BUDGETED COST PER WEEK ($)
A	0	0	2	22,000	11,000
B	0	1	3	30,000	10,000
C	2	2	2	26,000	13,000
D	3	4	4	48,000	12,000
E	4	4	4	56,000	14,000
F	4	10	3	30,000	10,000
G	8	8	5	80,000	16,000
H	13	13	2	16,000	8,000
			Total	308,000	

Budgeted cost - Earliest times

ACTIVITY	WEEK 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
A	11	11														22
B	10	10	10													30
C			13	13												26
D				12	12	12	12									48
E					14	14	14	14								56
F					10	10	10									30
G									16	16	16	16	16			80
H														8	8	16
																308
Total per week	21	21	23	25	36	36	36	14	16	16	16	16	16	8	8	
Total to date	21	42	65	90	126	162	198	212	228	244	260	276	292	300	308	

Budgeted cost - Latest times

ACTIVITY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
A	11	11														22
B		10	10	10												30
C			13	13												26
D					12	12	12	12								48
E					14	14	14	14								56
F											10	10	10			30
G									16	16	16	16	16			80
H														8	8	16
																308
Total per week	11	21	23	23	26	26	26	26	16	16	26	26	26	8	8	
Total to date	11	32	55	78	104	130	156	182	198	214	240	266	292	300	308	

Budget ranges