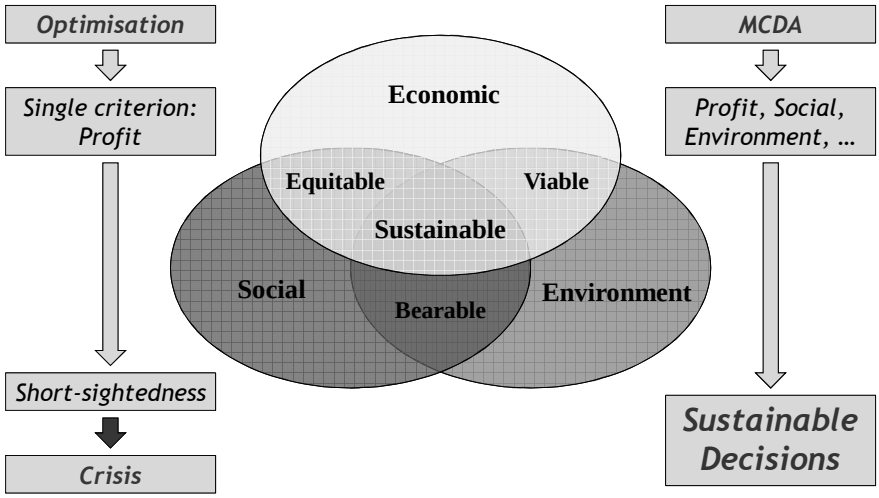
Summary

1. Introduction
   - Statistics, management science, operations research, decision aid, …
2. Advanced optimization
   - Linear programming
   - Integer programming
   - Non-linear programming
3. Multicriteria decision aid
4. Networks
   - Transportation problems
   - Network flow problems
   - Project management
5. Inventory management
6. Simulation models
7. Advanced statistical methods
   - Data mining
   - Analysis of variance
   - Forecasting

Some Evaluation or Decision Problems

- Location problems: plant, store, warehouse...
- Human resources management.
- Purchase of equipment.
- Evaluation of suppliers (quality).
- Project evaluation.
- Investment selection.

MCDA vs Optimisation



Multicriteria vs Unicriterion Models

- Unicriterion model:

$$\text{Optimise}\{g(a) | a \in A\}$$

  - Mathematically well stated:
    - Notion of optimal solution,
    - Complete ranking of actions.
  - Socio-economically ill stated:
    - Single criterion? Not so realistic.
    - Perception thresholds, …

Multicriteria vs Unicriterion Models

- Multicriteria model:

$$\text{Optimise}\{g_1(a), g_2(a), \ldots, g_k(a) \mid a \in A\}$$

  - Mathematically ill stated:
    - No optimal solution,
    - No mathematical meaning.
  - Socio-economically well stated:
    - Closer to actual decision problems,
    - Looking for a best compromise solution.

Multicriteria Table

- Actions:
  - possible decisions,
  - items to evaluate.
- Criteria:
  - quantitative,
  - qualitative.

Multicriteria Table

Action 1					
Action 2					
Action 3					
Action 4					
Action 5					
…					

Multicriteria Table

	Crit. 1 (unit)	Crit. 2 (unit)	Crit. 3 (unit)	Crit. 4 (unit)	…
Action 1					
Action 2					
Action 3					
Action 4					
Action 5					
…					

Multicriteria Table

	Crit. 1 (/20)	Crit. 2 (note)	Crit. 3 (apprec.)	Crit. 4 (Y/N)	…
Action 1	18	135	G	Yes	…
Action 2	9	147	B	Yes	…
Action 3	15	129	VG	No	…
Action 4	12	146	VB	?	…
Action 5	7	121	G	Yes	…
…	…	…	…	…	…

Plant Location

	Investment (BEF)	Costs (BEF)	Environn. (estimate)	…
Site 1	18	135	G	…
Site 2	9	147	B	…
Site 3	15	129	VG	…
Site 4	12	146	VB	…
Site 5	7	121	G	…
…	…	…	…	…

Purchase

	Price (BEF)	Reliability (days)	Maintenance (estimate)	…
Product A	18	135	G	…
Product B	9	147	B	…
Product C	15	129	VG	…
Product D	12	146	VB	…
Product E	7	121	G	…
…	…	…	…	…

An Example

Purchase of an automobile

Objectives:

- Purchase (price),
- Use (fuel consumption),
- Performance (power),
- Comfort,
- Space.

Multicriteria Table

Car	Price	Power	Consumpt	Space	Comfort
Tourism A	360000	75	8,0	3	3
Sport	390000	110	9,0	1	2
Tourism B	355000	85	7,0	4	3
Luxury 1	480000	90	8,5	4	5
Economic	250000	50	7,5	2	1
Luxury 2	450000	85	9,0	5	4

- What is the best buy?

Multicriteria Table

Car	Price	Power	Consumpt	Space	Comfort
Tourism A	360000	75	8,0	3	3
Sport	390000	**110**	9,0	1	2
Tourism B	355000	85	**7,0**	4	3
Luxury 1	480000	90	8,5	4	**5**
Economic	**250000**	50	7,5	2	1
Luxury 2	450000	85	9,0	**5**	4

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Multicriteria Table

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Tourism B	355000	85	**7,0**	4	3
Luxury 1	480000	90	8,5	4	**5**
Economic	**250000**	50	7,5	2	1
Luxury 2	450000	85	9,0	**5**	4

- What is the best buy?
- What is the best compromise?

Multicriteria Table

Car	Price	Power	Consumpt	Space	Comfort
Tourism A	360000	75	8,0	3	3
Sport	390000	**110**	9,0	1	2
Tourism B	355000	85	**7,0**	4	3
Luxury 1	480000	90	8,5	4	**5**
Economic	**250000**	50	7,5	2	1
Luxury 2	450000	85	9,0	**5**	4

- What is the best buy?
- What is the best compromise?
- What are the priorities of the buyer?

Preference Modelling

- Problem:

  How to compare two actions
  a and *b* ?

- A first model: 3 possible situations:
  1. Preference: aPb or bPa
  2. Indifference: aIb
  3. Incomparability: aRb

Preference structure

- Properties (logical):

$aPb \Rightarrow$ not bPa	P is asymmetrical
aIa	I is reflexive
$aIb \Rightarrow bIa$	I is symmetrical
Not aRa	R is irreflexive
$aRb \Rightarrow bRa$	R is symmetrical

- These three relations define a preference structure (s.p.), if for all *a*,*b* in A we always have on of the following situations:

  aPb or bPa or aIb or aRb

Traditional preference structure (unicriterion)

- Optimisation of a function g on A

$$\forall a,b \in A: \begin{cases} aPb & \Leftrightarrow & g(a) > g(b) \\ aIb & \Leftrightarrow & g(a) = g(b) \end{cases}$$

- Consequences :

R is empty
P is transitive
I is transitive

- Complete ranking (weak order).

Introduction of an indifference threshold

- Problem: Intransitivity of indifference.
  Cf. Paradox of the coffee cup (Luce, 1956)
- Introduction of the indifference threshold:

$$\forall a,b \in A: \begin{cases} aPb & \Leftrightarrow & g(a) > g(b) + q \\ aIb & \Leftrightarrow & |g(a) - g(b)| \leq q \end{cases}$$

- Quasi-order: P is transitive, but I is not.

Some other preference structures

- Variable indifference threshold
  $\Rightarrow$ Interval order.
- Indifference threshold + preference threshold
  $\Rightarrow$ Pseudo-order.
- Models with incomparability
  $\Rightarrow$ Partial orders.
- Valued preference structures.

Social choice theory

- Problem:
  - A group of voters have to select a candidate among a group of candidates (election).
  - Each voter has a personal ranking of the candidates according to his/her preferences.
  - Which candidate should be elected?
- What is the « best » voting procedure?
- Analogy with multicriteria models:
  - Candidates $\leftrightarrow$ actions,
  - Voters $\leftrightarrow$ criteria.

5 procedures…
… among many others…

1. Relative majority.
2. Condorcet.
3. Second ballot (French presidential).
4. Borda.
5. Successive eliminations.

Procedure 1 : Relative majority

3 candidates: **A**lbert**, B**runo**, C**laire

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

A	11
B	10
C	9

Albert is elected

Procedure 1 : Relative majority

3 candidates: **A**lbert**, B**runo**, C**laire

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

A	11
B	10
C	9

Problem: B and C preferred to A by a majority of voters!

Albert is elected

Marie Jean Antoine Nicolas de Caritat Marquis de Condorcet 1743 - 1794

Procedure 2 : Condorcet

3 candidates: **A**lbert**, B**runo**, C**laire

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

B preferred to **A**	19 votes
B preferred to **C**	21 votes
C preferred to **A**	19 votes

Bruno is elected

Procedure 2 : Condorcet paradox

3 candidates: **A**lbert**, B**runo**, C**laire

9 voters:

4 voters	**3** voters	**2** voters
A	B	C
B	C	A
C	A	B

A preferred to **B**	6 votes
B preferred to **C**	7 votes
C preferred to **A**	5 votes

Nobody is elected!

Procedure 3 : second ballot
(French presidential election)

4 candidates: **A**lbert**, B**runo**, C**laire, **D**iane

63 voters:

22 voters	**21** voters	**20** voters
B	C	D
A	A	A
C	D	C
D	B	B

1st tour:	**B** and **C**
2nd tour:	**C** beats **B** (41 vs 22)

Claire is elected

Procedure 3 : second ballot
(French presidential election)

4 candidates: **A**lbert**, B**runo**, C**laire, **D**iane **63** voters:

22 voters	**21** voters	**20** voters
B	C	D
A	A	A
C	D	C
D	B	B

Claire is elected **!!!**

...but

A preferred to **C**	42 votes
A preferred to **B**	41 votes
A preferred to **D**	43 votes

Procedure 3 : second ballot
(French presidential election)

3 candidates: **A**lbert**, B**runo**, C**laire
17 voters:

5 voters	6 voters	4 voters	2 voters
C	A	B	B
A	B	C	A
B	C	A	C

1st tour:	**A** and **B**
2nd tour:	**A** beats B (11 vs 6)

Albert is elected

Procedure 3 : second ballot
(French presidential election)

3 candidates: **A**lbert**, B**runo**, C**laire
17 voters:

Albert was elected

5 voters	6 voters	4 voters	2 voters
C	A	B	~~A~~ B
A	B	C	B ~~A~~
B	C	A	C

1st tour:	**A** and **C**
2nd tour:	**C** beats A (9 to 8)

Claire is elected !

Problem: non-monotonicity!

Jean Charles de Borda
1733 - 1799

Procedure 4 : Borda

3 candidates: **A**lbert**, B**runo**, C**laire

81 voters:

30 voters	**29** voters	**10** voters	**10** voters	**1** voter	**1** voter
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points	Score	
2	A	**101**
1	B	**33**
0	C	**109**

31 x 2 + 39 x 1

11 x 2 + 11 x 1

39 x 2 + 31 x 1

Claire is elected!

Procedure 4 : Borda

3 candidates: **A**lbert**, B**runo**, C**laire

81 voters:

30 voters	**29** voters	**10** voters	**10** voters	**1** voter	**1** voter
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points
2
1
0

Scores	
A	**101**
B	**33**
C	**109**

A preferred to **C : 41** on 81

Procedure 4 : Borda

4 candidates: **A**lbert**, B**runo**, C**laire, **D**iane

7 voters:

3 voters	**2** voters	**2** voters	**Points**
C	B	A	3
B	A	D	2
A	D	C	1
D	C	B	0

Scores	
A	13
B	12
C	11
D	6

Ranking
A
B
C
D

Albert is elected

Procedure 4 : Borda

4 candidates: **A**lbert**, B**runo**, C**laire, **D**iane

7 voters:

3 voters	**2** voters	**2** voters	**Points**
C	B	A	2
B	A	C	1
A	C	B	0

Scores	
A	6
B	7
C	8

Ranking
C
B
A

Claire is elected

Borda
(manipulation)

3 candidates: **A**lbert**, B**runo**, C**laire

34 voters:

Bruno's partisans generate the candidacy of x (« fake candidate »)

Scores	
A	46
B	36
C	20

Ranking
A
B
C

Albert is elected

Borda
(manipulation)

4 candidates: **A**lbert**, B**runo**, C**laire, **x**

34 voters:

12 voters	**12** voters	**10** voters	**Points**
A	B	C	3
B	x	A	2
C	A	B	1
x	C	x	0

Scores	
A	68
B	70
C	42
x	24

Ranking
B
A
C
x

Bruno is elected!

Borda
(manipulation)

4 candidates: **A**lbert**, B**runo**, C**laire, **x**

34 voters:

12 voters	**12** voters	**10** voters	**Points**
A	B	C	3
x	x	x	2
B	A	A	1
C	C	B	0

Scores	
A	58
B	48
C	30
x	68

Ranking
x
A
B
C

The fake candidate is elected!

Procedure 5 : Successive eliminations

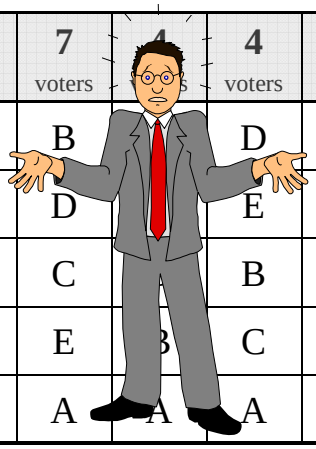
- Tour-wise procedure.
- Principle:
  Eliminate progressively the worst candidates, one by one, until only one is left.

Conclusion?

5 candidates: **A**lbert**, B**runo**, C**laire, **D**iane, **E**ric

25 voters:

8 voters	**7** voters	**4** voters	**4** voters	**2** voters
A	B	D	D	C
C	D	E	E	E
D	C	B	B	D
B	E	C	C	B
E	A	A	A	A



Relative majority
→ **Albert** elected

Second ballot:
→ **Bruno** elected

Condorcet:
→ **Claire** elected

Borda:
→ **Diane** elected

Successive eliminations:
→ **Eric** elected

Kenneth Arrow
(Nobel prize in economy, 1972)

- Impossibility theorem (1952):
  With at least 2 voters and 3 candidates, it is **impossible** to build a voting procedure that simultaneously satisfies the 5 following properties:
  - Non-dictatorship.
  - Universality.
  - Independence with respect to third parties.
  - Monotonicity.
  - Non-imposition.

A common approach:
The weighted sum

Criteria

	g_1	g_2	g_3	...
a	$g_1(a)$	$g_2(a)$	$g_3(a)$	...
b	$g_1(b)$	$g_2(b)$	$g_3(b)$	...
c	...			
...	...			
	w_1	w_2	w_3	...

Actions or Decisions

Weights of the criteria

A common approach:
The weighted sum

- Global value for a :

  $V(a) = w_1\, g_1(a) + w_2\, g_2(a) + \ldots$

- a is preferred to b if:

  $V(a) > V(b)$

  (if all criteria are to maximise)

Weighted sum:
Example 1

	g_1	g_2	g_3	g_4	g_5
a	100	100	100	100	55
b	85	85	85	85	100
	1/5	1/5	1/5	1/5	1/5

- $V(a) = 91 \qquad V(b) = 88$
- Total and uncontrolled compensation of weaknesses by strengthes.

Weighted sum:
Example 2

	g_1	g_2
a	100	0
b	0	100
c	50	50
d	50	50
	1/2	1/2

- V(*a*) = V(*b*) = V(*c*) = V(*d*) = 50
- Elimination of conflicts – Loss of information.

Weighted sum:
Example 3

"Profit is approximately 2 times more important than time savings;
0.7 for profit and 0.3 for time savings."

	g_1 (BF)	g_2 (min)
a	60	60
b	48	70
	0.7	0.3

V(*a*) = 60
V(*b*) = 54.6

a is ranked 1st.

Weighted sum:
Example 3

"Profit is approximately 2 times more important than time savings;
0.7 for profit and 0.3 for time savings."

	g_1 (FF)	g_2 (min)
a	10	60
b	8	70
	0.7	0.3

V(*a*) = 25
V(*b*) = 26.6

b is ranked 1st!

Weighted sum:
Example 3

	g_1 (BF)	g_2 (min)
a	60	60
b	48	70
	0.7	0.3

V(*a*) = 60
V(*b*) = 54.6

a is ranked 1st.

	g_1 (FF)	g_2 (min)
a	10	60
b	8	70
	0.7	0.3

V(*a*) = 25
V(*b*) = 26.6

b is ranked 1st.

 → Significance of "weights" ! ←

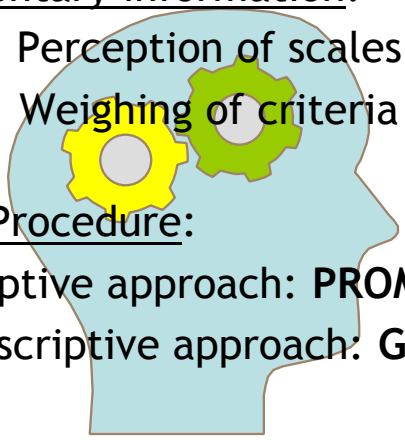
Multicriteria decision aid

- Multiattribute utility theory.
- Outranking methods.
- Interactive methods.
- Multiobjective programming.
- …

Since 1970, numerous developments: conferences, papers, books, applications, software...

Decision aid methods

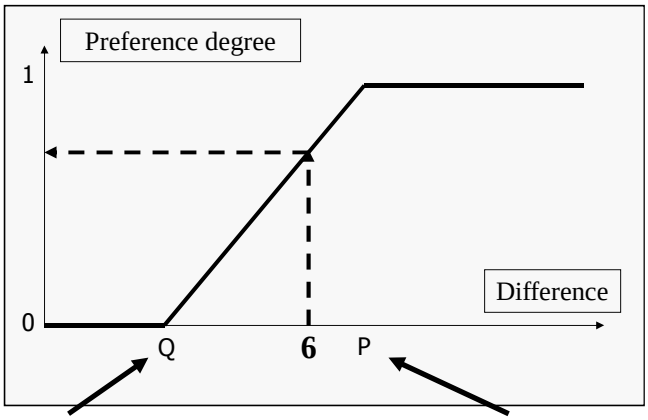
- Supplementary information:
  Perception of scales
  Weighing of criteria
- Analysis Procedure:
  Prescriptive approach: **PROMETHEE**
  Descriptive approach: **GAIA**



Comparison of 2 actions

	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	…
Action 1	18	135	B	Oui	…
Action 2	9	147	Difference = 6		…
Action 3	15	129	TB	Non	…
Action 4	12	146	TM	?	…
Action 5	7	121	B	Oui	…
…	…	…	…	…	…

Preference function



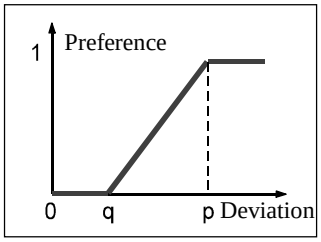
Indifference threshold — Linear — Preference threshold

PROMETHEE

	Economic		Lux. 1	
-230000	250000	*Price*	480000	
	50	*Power*	90	+40
-1,0	7,5	*Consumpt.*	8,5	
	B	*Space*	G	+2
	VB	*Comfort*	VG	+4

PROMETHEE

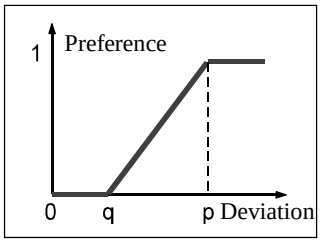
		Economic		Lux. 1		
1,0	-230000	250000	*Price*	480000		
		50	*Power*	90	+40	1,0
0,5	-1,0	7,5	*Consumpt.*	8,5		
		B	*Space*	G	+2	0,5
		VB	*Comfort*	VG	+4	1,0



PROMETHEE

Pref (Eco.,Lux.)

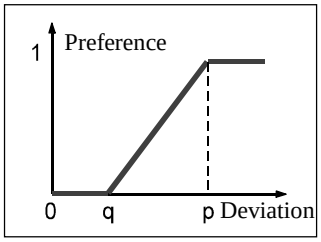
		Economic		Lux. 1		**Pref (Lux.,Eco.)**
1,0	**-230000**	250000	*Price*	480000		0,0
0,0		50	*Power*	90	**+40**	**1,0**
0,5	**-1,0**	7,5	*Consumpt.*	8,5		0,0
0,0		B	*Space*	G	**+2**	**0,5**
0,0		VB	*Comfort*	VG	**+4**	**1,0**



PROMETHEE

Pref (Eco.,Lux.) **Pref (Lux.,Eco.)**

		Economic		Lux. 1			*Wght*
1,0	**-230000**	250000	*Price*	480000		0,0	*1*
0,0		50	*Power*	90	**+40**	**1,0**	*1*
0,5	**-1,0**	7,5	*Consumpt.*	8,5		0,0	*1*
0,0		B	*Space*	G	**+2**	**0,5**	*1*
0,0		VB	*Comfort*	VG	**+4**	**1,0**	*1*

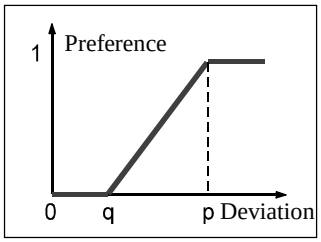


- Pref (Eco.,Lux.) = 0,3
  = (1 + 0 + 0,5 + 0 + 0) / 5
- Pref (Lux.,Eco.) = 0,5
  = (0 + 1 + 0 + 0,5 + 1) / 5

PROMETHEE

Pref (Eco.,Lux.) **Pref (Lux.,Eco.)**

		Economic		Lux. 1			Wght
1,0	**-230000**	250000	*Price*	480000		0,0	*2*
0,0		50	*Power*	90	**+40**	**1,0**	*1*
0,5	**-1,0**	7,5	*Consumpt.*	8,5		0,0	*2*
0,0		B	*Space*	G	**+2**	**0,5**	*1*
0,0		VB	*Comfort*	VG	**+4**	**1,0**	*1*



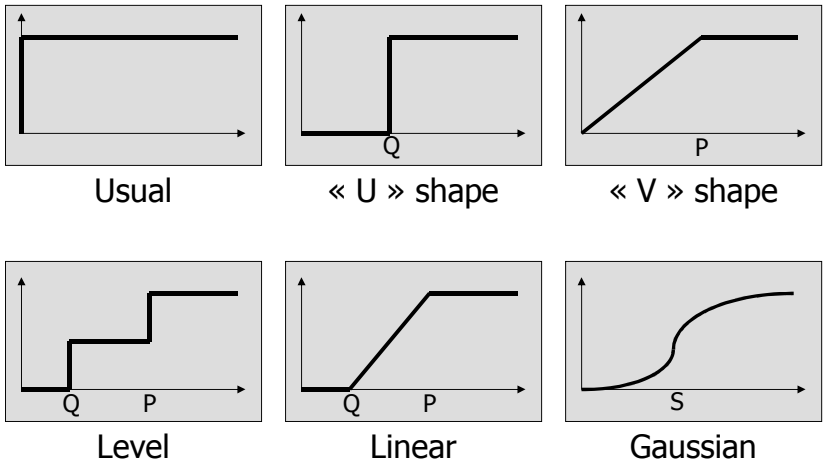
- Pref (Eco.,Lux.) = 0,43
  = (2 x 1 + 0 + 2 x 0,5 + 0 + 0) / 7
- Pref (Lux.,Eco.) = 0,36
  = (0 + 1 + 0 + 0,5 + 1) / 7

Pairwise comparisons

- For each criterion g_j :
  - Preference function P_j
  - Weight w_j
- Multicriteria preference degree of a over b :

$$\pi(a,b) = \sum_{j=1}^{k} w_j \; P_j(a,b)$$

Preference functions
(as in Decision Lab software)

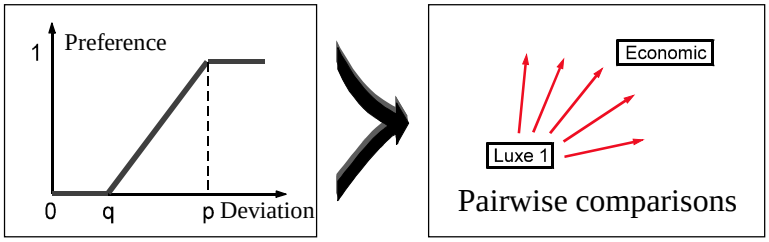


Usual « U » shape « V » shape

Level Linear Gaussian

PROMETHEE

Pref (Eco.,Lux.)	Economic		Lux. 1	Pref (Lux.,Eco.)
1,0	**-230000**	250000 *Price*	480000	0,0
0,0		50 *Power*	90	**+40**	**1,0**
0,5	**-1,0**	7,5 *Consumpt.*	8,5	0,0
0,0		B *Space*	G	**+2**	**0,5**
0,0		VB *Comfort*	VG	**+4**	**1,0**



Pairwise comparisons

Pairwise preference matrix $\pi(a,b)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00						
Sport		0,00					
Tour.B			0,00				
*Lux.*1				0,00	0,50		
Econ.				0,30	0,00		
*Lux.*2						0,00	
$\phi^-(a)$							
$\phi(a)$							

Pairwise preference matrix $\pi(a,b)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	
Sport	0,20	0,00	0,16	0,24	0,30	0,24	
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	
$\phi^-(a)$							
$\phi(a)$							

Computation of $\phi^{+}(a)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^{+}(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^{-}(a)$							
$\phi(a)$							

Computation of $\phi^{+}(a)$

$\pi(a,b)$	*Moy.A*	*Sport*	*Moy.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^{+}(a)$
Moy.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Moy.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^{-}(a)$							
$\phi(a)$							

Computation of $\phi^-(a)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$							

Computation of $\phi^-(a)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$							

Computation of $\phi(a)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$	0,02	-0,17	0,26	0,06	-0,15	0,00	

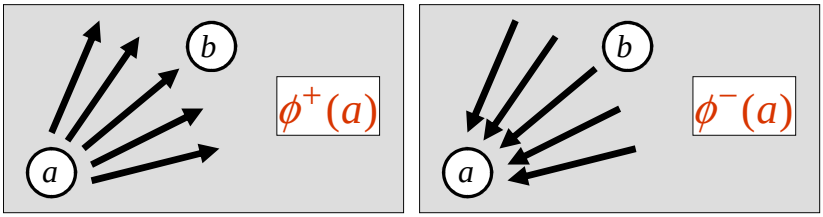
Computation of $\phi(a)$

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$	0,02	-0,17	0,26	0,06	-0,15	0,00	

Computation of preference flows

$\pi(a,b)$	*Tour.A*	*Sport*	*Tour.B*	*Lux.*1	*Econ.*	*Lux.*2	$\phi^+(a)$
Tour.A	0,00	0,34	0,00	0,21	0,26	0,22	0,21
Sport	0,20	0,00	0,16	0,24	0,30	0,24	0,23
Tour.B	0,15	0,55	0,00	0,32	0,45	0,33	0,36
*Lux.*1	0,18	0,45	0,10	0,00	0,50	0,15	0,28
Econ.	0,20	0,34	0,14	0,30	0,00	0,35	0,27
*Lux.*2	0,24	0,30	0,10	0,04	0,60	0,00	0,26
$\phi^-(a)$	0,19	0,40	0,10	0,22	0,42	0,26	
$\phi(a)$	0,02	-0,17	0,26	0,06	-0,15	0,00	

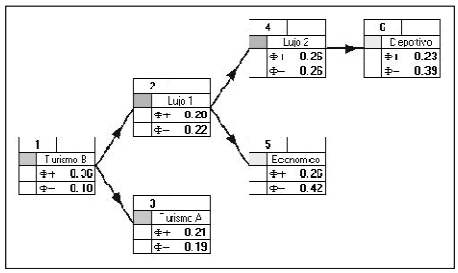
Preference flows

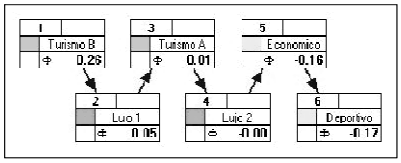


- Leaving flow: (strength) $\phi^+(a) = \frac{1}{n-1} \sum_{b \in A} \pi(a,b)$
- Entering flow: (weakness) $\phi^-(a) = \frac{1}{n-1} \sum_{b \in A} \pi(b,a)$
- Net flow: $\phi(a) = \phi^+(a) - \phi^-(a)$

PROMETHEE

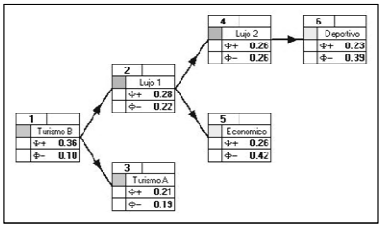
- Rank decisions from the best to the worst ones.
- Identify best compromise solutions.

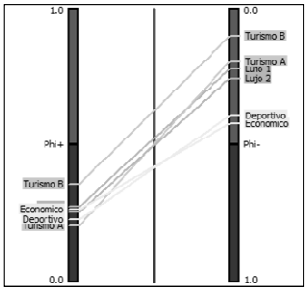




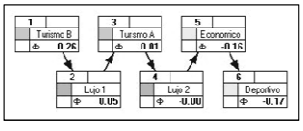
PROMETHEE I & II

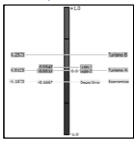
- PROMETHEE I : partial ranking – ϕ^+, ϕ^-



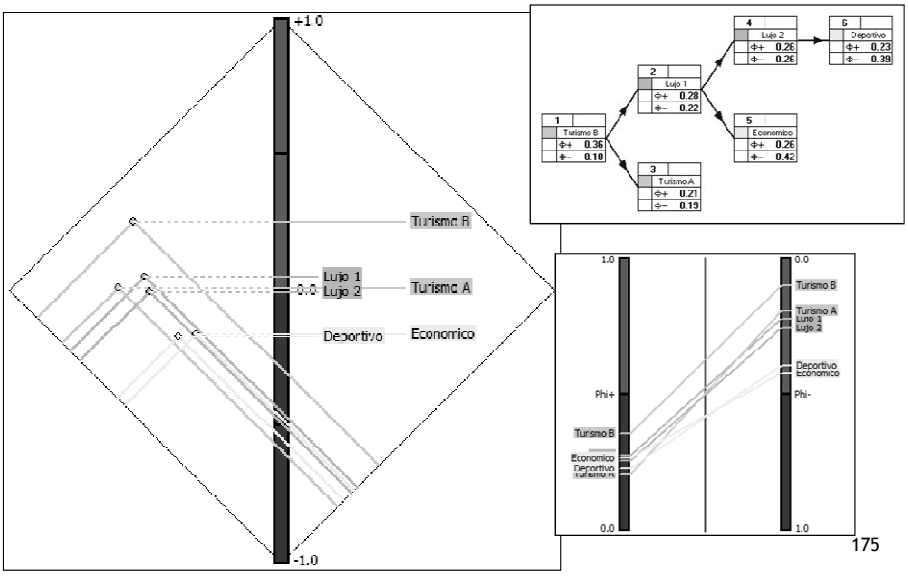


- PROMETHEE II : complete ranking – ϕ





PROMETHEE I Diamond



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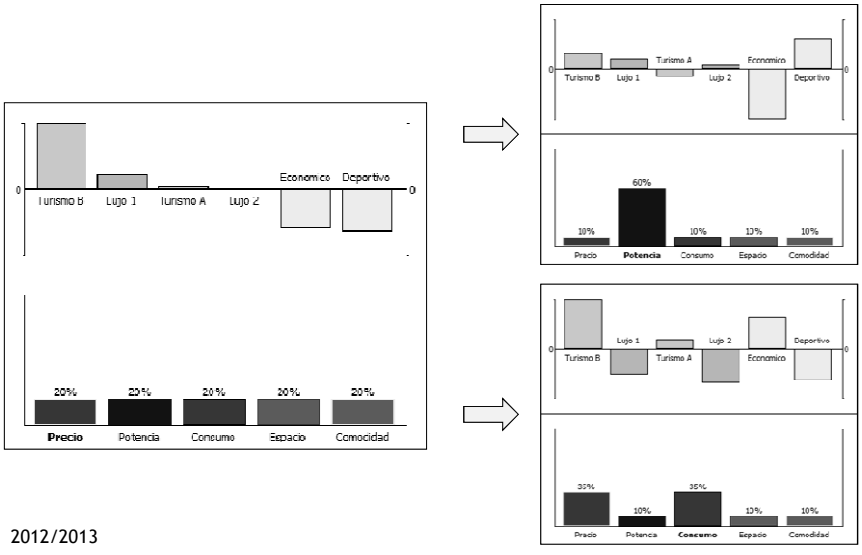
Sensitivity Analysis with PROMETHEE

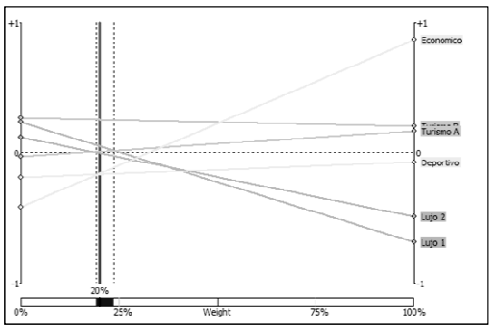
- Criteria weights $\leftrightarrow$ PROMETHEE ranking.
- Interactive weight sensitivity analysis: « Walking Weights ».
- Robustness with respect to weight values?
  - Weight stability intervals.
  - Visual weight stability intervals.

Walking Weights

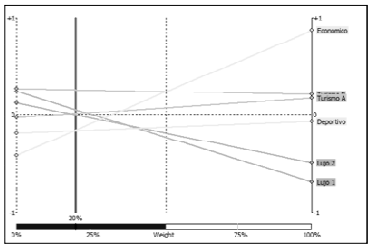


2012/2013

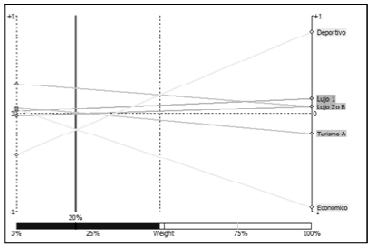
Visual Stability Intervals



VSI for « Precio » (level 6):
[19.20% , 23.70%]



VSI for « Precio » (level 1): [0.00% , 50.68%]



VSI for « Potencia » (level 1): [0.00% , 48.65%]

2012/2013

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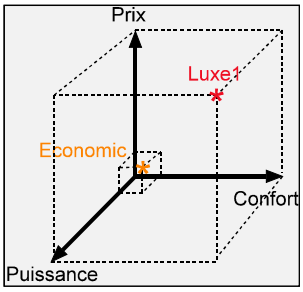
GAIA

- Visual descriptive analysis.
- Better understanding:
  - Conflicting criteria.
  - Action profiles.
  - Possible compromise solutions.
- Reducing the multicriteria dimension:
  - Principal components analysis.

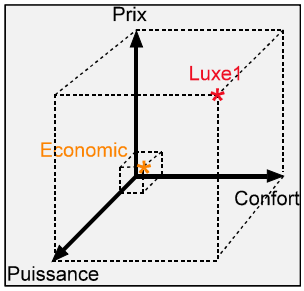
GAIA

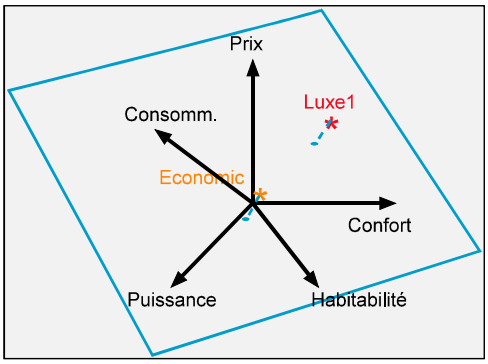


- Graphical representation.
- 5 dimensions!

GAIA

1. ***Computation of unicriterion net flows (normalization)***
2. ***Projection on a plane:***

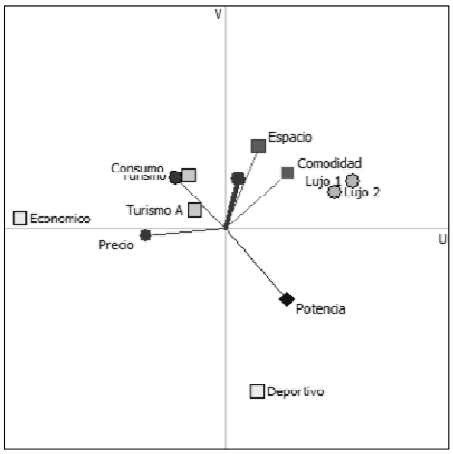




- Graphical representation.
- 5 dimensions!

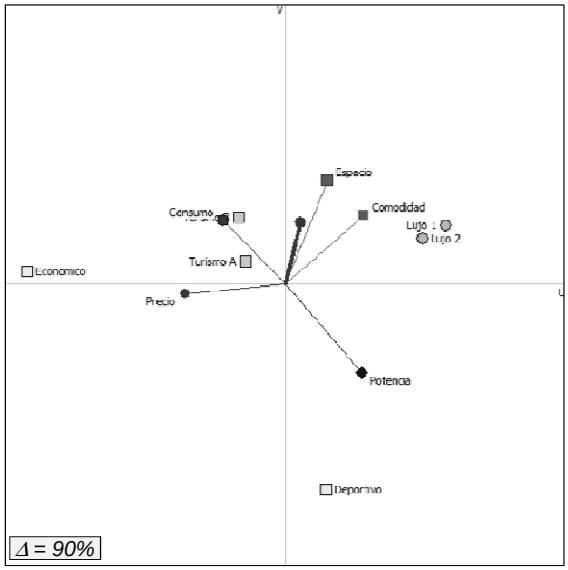
GAIA

- Discover conflicts among criteria.
- Identify potential compromises.
- Help to fix priorities.



GAIA

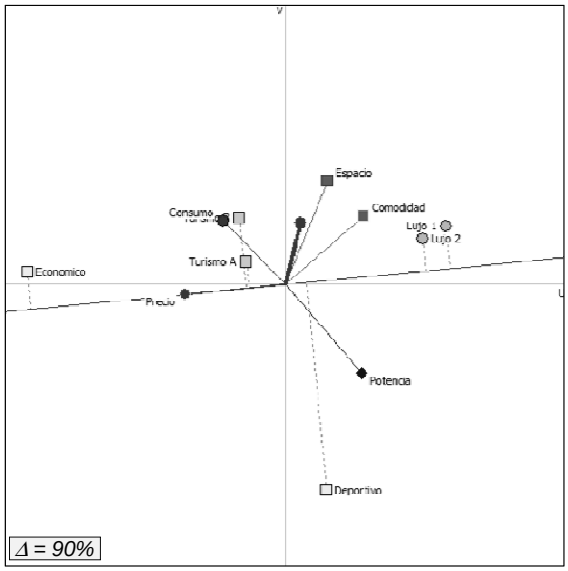
- *Actions: points*
- *Criteria: axes*



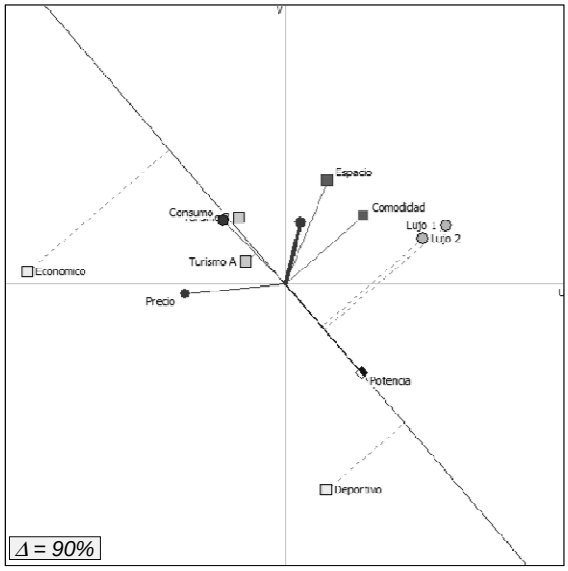
GAIA

Precio

- *Economico: 15 k€*
- *Turismo: 25,5-26 k€*
- *Deportivo: 29 k€*
- *Lujo: 35-38 k€*



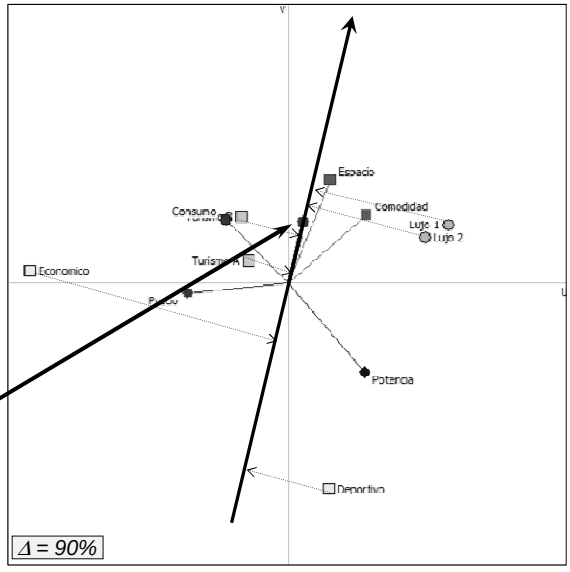
GAIA



Potencia

- *Deportivo: 110 kW*
- *Lujo: 85-90 kW*
- *Turismo: 75-85 kW*
- *Economico: 50 kW*

GAIA

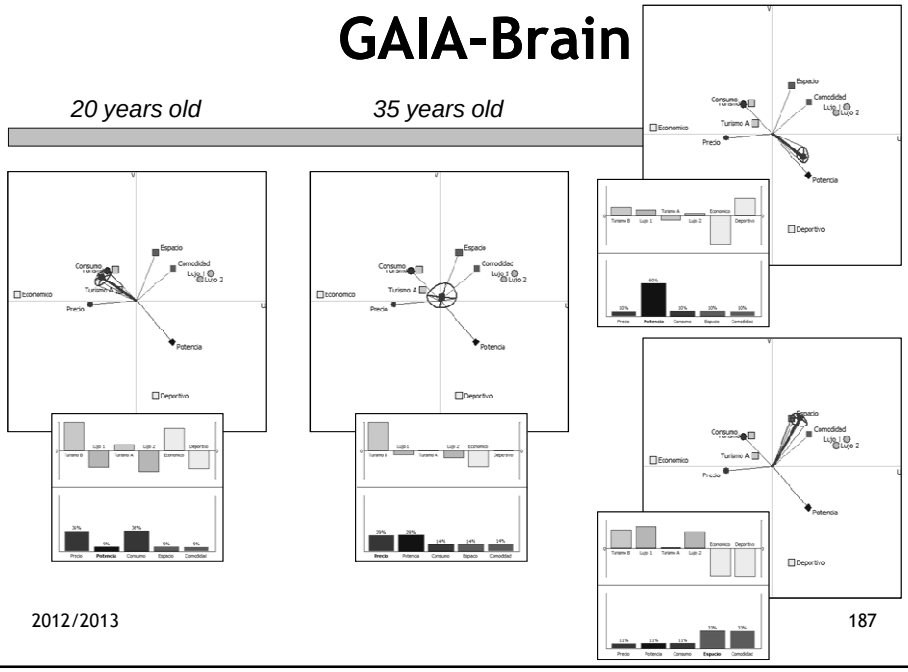


PROMETHEE II !

- *Turismo B : 0,26*
- *Lujo 1 : 0,06*
- *Turismo A : 0,02*
- *Lujo 2 : 0,00*
- *Economico : -0,15*
- *Deportivo : -0,17*

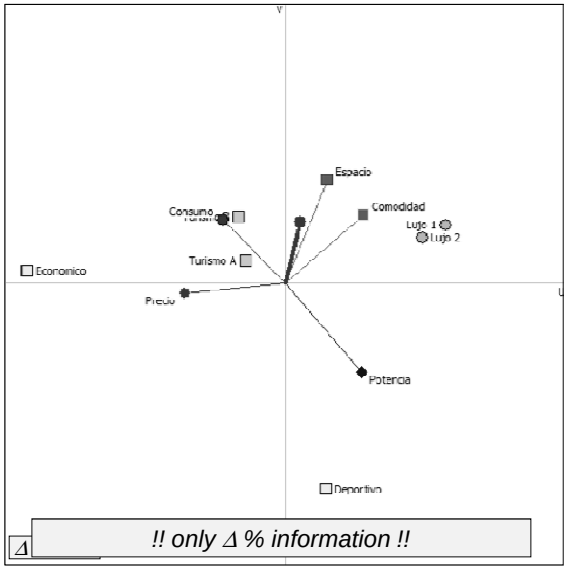
GAIA-Brain

20 years old *35 years old*



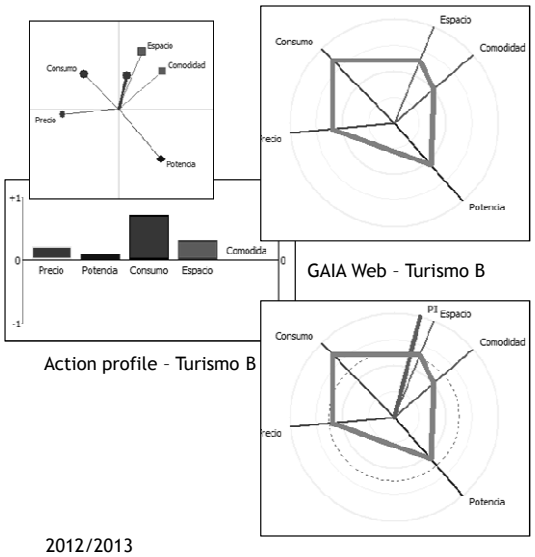
GAIA

- *Actions: points*
- *Criteria: axes*
- *Decision axis*



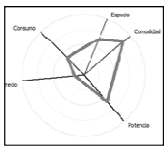
!! only Δ % information !!

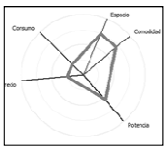
GAIA Webs

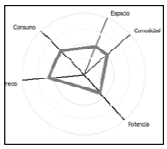


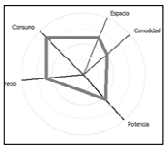
Action profile – Turismo B

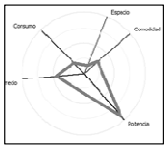
GAIA Web – Turismo B

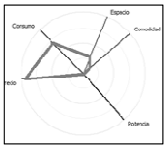












Example 2 : Plant location

- Actions: 5 potential sites
- Criteria:
  - g_1 : Cost (investment)
  - g_2 : Cost (operations)
  - g_3 : Employment
  - g_4 : Transportation
  - g_5 : Environmental impact
  - g_6 : Social impact

Evaluation table

Tutorial2

	Investment	Operations	Employment	Transportation	Environment	Social
Min/Max	Minimize	Minimize	Minimize	Maximize	Minimize	Minimize
Weight	25.0000	15.0000	20.0000	20.0000	10.0000	10.0000
Preference Functi	Linear	Linear	Linear	Level	Level	Level
Indifference Thres	5.00 %	5.00 %	5.00 %	0.5000	0.5000	0.5000
Preference Thres	25.00 %	25.00 %	10.00 %	1.5000	1.5000	1.5000
Gaussian Thresh						
Threshold Unit	Percent	Percent	Percent	Absolute	Absolute	Absolute
Unit	M$	M$	workers	5-point	Impact	Impact
Site 1	74.0000	12.0000	175.0000	Average	High	Low
Site 2	86.0000	9.0000	170.0000	Good	Low	Very Low
Site 3	89.0000	7.0000	145.0000	Very Good	Very Low	Moderate
Site 4	115.0000	8.0000	95.0000	Bad	Low	High
Site 5	128.0000	10.0000	110.0000	Good	Moderate	Very Low

Industrial / Political Environment / Social /

- Criteria to minimize or maximize.
- Different scales.
- Quantitative or qualitative criteria.

Single- and Multi-decision maker decision problems

- Single-decision maker :
  - Single stakeholder (decision maker).
  - Single evaluation table and preference structure.
- Multi-decision maker:
  - Multiple stakeholders (including decision maker(s)).
  - Multiple evaluation tables and preference structures.
  - Looking for a consensus solution.

Example 2

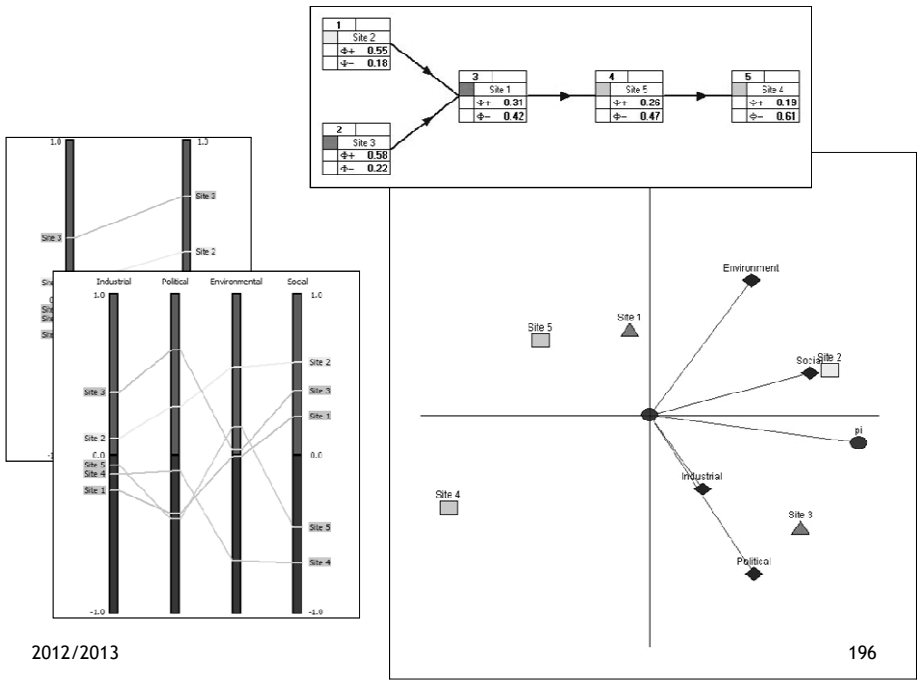
- Four stakeholders ("decision makers"):
  - Industrial (actual decision maker),
  - Political authorities (regional),
  - Environmental protection groups,
  - Workers unions (social).
- Four multicriteria tables.

Multi-scenarios model

- Scenarios:
  - Points of view,
  - Hypotheses, …
- Evaluations:
  - 'Objective' criteria: common evaluations.
  - 'Subjective' criteria: specific evaluations for each scenario.
- Specific preference structures :
  - Weights, preference thresholds.

Multi-scenarios model

- Adaptation of PROMETHEE:
  - Individual rankings.
  - Global (group) rankings taking into account a possible weighing of the scenarios.
- Adaptation of GAIA:
  - Two distinct analyses.



Visual PROMETHEE

WWW.PROMETHEE-GAIA.NET

- 3-level simple hierarchical criteria structure.
- New visual tools:
  - PROMETHEE rankings and Diamond,
  - Visual Weight Stability Intervals,
  - Decision-maker's Brain (PROMETHEE VI),
  - GAIA-3D,
  - GAIA-Webs and PROMap GIS integration,
  - Performance (input-output) analysis, …

PROMap

- *Google Maps* integration:

Group decision making

- Up to 80% of upper management and executives working time spent in meetings.
  - Time consuming (meetings, travel),
  - High cost.
- Limited efficiency of classical meetings:
  - Limited time allocated to each participant,
  - Psychological restraints,
  - Limited memory, …
- Important stakes for organisations.

GDSS Rooms

Group Decision Support System

- Use IT to improve the efficiency of meetings.
  - Electronic brainstorming.
    - Working in parallel.
    - Possible anonymity.
    - Automated report generation.
  - Decision Aid.
  - Voting procedures.
- GDSS rooms or Internet.
- Time savings and costs reduction.

Some applications at ULB

- Financial evaluation of companies.
- Quality assesment of suppliers.
- Electricity production planning at Electrabel.
- Regional planning.
- Evaluation of urban waste management systems.
- Environmental applications.
- Therapeutical choice.
- ...